Multifractal analysis of flows in fully developed turbulence: applications to atmospheric entropy estimation and pluviometry

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Flows in fully developed turbulence exhibit invariances (in scale, translational) on the relevant statistical variables. On the other hand, those flows contain intermittent areas, over which activity is neglicible at a given observation scale. For that reason they cannot be characterized by a single self-similarity exponent (the simplest scale transformation), but by a collection of exponents connected to local anisotropies. They are not thus fractals (a single scaling law does not define them) but multifractals (several scaling laws coexist in the flow). In order to describe such systems the concept of statistical self-similarity was introduced. Statistical self-similarity allows to characterize turbulent flows by the exponents derived from a certain probability distribution. This approach provides the so-called Rényi's generalized dimensions[1], which constitute an evidence of multifractal behaviour (multiple dimensions associated to multiple fractal components). This characterization is rather poor however, as it does not allow extracting the actual fractal components.

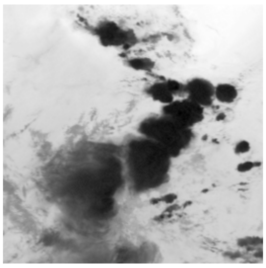

FIG. 1. Original IR image

Singularity analysis[2] provides a true geometrical description of the flow, in which every point is assigned to a fractal component regarding the value of its singularity exponent. In this work we have developed this technique, stressing its physical meaning for the particular case of MetoSat infrared (IR) images (figure 1). Such images give a measure of the temperature on the highest layers of atmosphere and are thus expected to be multifractal, as temperature is a meaningful thermodynamical variable. The goal of this study is to locate precipitant areas from IR records, as they are sampled at higher spatial and temporal rate than other bandwidth records from some non-geostationary satellites (which on the contrary allow direct location of precipitation).

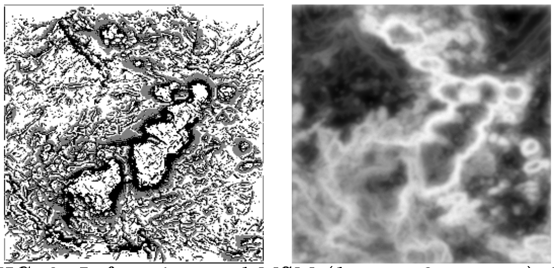

FIG. 2. Left: orientated MSM ($h_\infty = -0.5 \pm 0.35$); Right: Local entropy

Singularity analysis on the temperature field revealed that temperature is really multifractal, of the *log-Poisson* class. Log-Poisson multifractals are reconstructible from the value of the field over one fractal component, the Most Singular Manifold (MSM)[3]. The MSM is the most informative set, which is evidenced by the computation of the local entropy (a measure of the local amount of information). The entropy turned out to be locally maximal on the MSM (see figure 2). Finally, we made use of the technique of source extraction, which separates the geometry of the MSM from the distribution of temperatures over it. The resulting source field is interpreted as thermic bath with temperature dissipation and injection points. Comparing the source field with a direct pluviometry record from another satellite it could be observed a good correspondence between precipitation and thermal variation points (figure 3).

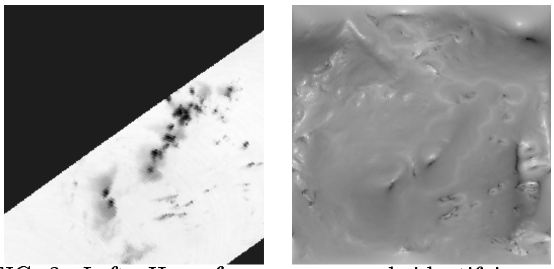

FIG. 3. Left: Hyperfrequency record, identifying precipitant areas. Right: temperature sources

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