Asymmetric coupling effects in the synchronization of spatially extended chaotic systems

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We analyze the effects of asymmetric couplings in setting different synchronization states for a pair of unidimensional fields obeying Complex Ginzburg Landau equations. Novel features such as asymmetry enhanced complete synchronization, limits for the appearance of phase synchronized states, and selection of the final synchronized dynamics are reported and characterized.

So far, studies on synchronization of chaos have mainly focused on external forcings, and bidirectional symmetric or unidirectional master-slave coupling schemes1. However, in nature we cannot expect to have purely unidirectional, nor perfectly symmetrical coupling configurations. Therefore, our intention is to address the effects of asymmetries in the coupling of space-extended continuous fields.

We will refer to a pair of unidimensional fields obeying the Complex Ginzburg-Landau Equations (CGLE).

\[ A_{1,2} = A_{1,2} + (1 + i\alpha) \partial_x^2 A_{1,2} - (1 + i\beta_{1,2}) | A_{1,2} |^2 A_{1,2} 
+ \frac{c}{2} (1 \mp \theta)(A_{2,1} - A_{1,2}). \]  

(1)

Here \( A_{1,2}(x,t) = \rho_{1,2}(x,t)e^{i\phi_{1,2}(x,t)} \) are two complex fields (of amplitudes \( \rho_{1,2}(x,t) \) and phases \( \phi_{1,2}(x,t) \)), dots denote temporal derivatives, \( \partial_x^2 \) stays for the second derivative with respect to the space variable \( 0 \leq x \leq L \), \( L \) is the system extension, \( \alpha \) and \( \beta_{1,2} \) are suitable real parameters, \( c \) represents the coupling strength and \( \theta \) is a parameter accounting for asymmetries in the coupling. Precisely, the case \( \theta = 0 \) recovers the bidirectional symmetric coupling configuration, whereas the case \( \theta = 1 (\theta = -1) \) recovers the unidirectional master slave scheme, with the field \( A_1(A_2) \) driving the response of \( A_2(A_1) \).

We will discuss how to characterize the synchronization properties of the coupled fields by means of suitable indicators. The synchronization threshold and synchronized states will depend crucially on the asymmetry. Another effect of asymmetry is the transition from normal to anomalous frequency synchronized (FS) states.

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