Characterization of the chaos generated by semiconductor lasers subject to electro-optical and all-optical feedback

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We characterize the chaotic dynamics of semiconductor lasers subject to either optical or electro-optical feedback. This characterization is relevant for secure optical communications based on chaos encryption. In particular, we compute as function of system parameters the following quantifiers: Lyapunov spectrum (which mesaures the rate at which the distance between infinitesimally close solutions increases in time), the Kaplan-Yorke dimension (conjectured to be equal to the information dimension which measures the amount of information needed to locate the system in phase space with infinitesimal accuracy) and the Kolmogorov-Sinai entropy (which mesaures the average loss of information rate, or equivalently is inversely proportional to the time interval over which the future evolution can be predicted).

We consider a model for a laser with electro-optical feedback as described in[1], which is basically an Ikeda delayed equation. For the case of optical feedback we use the Lang-Kobayashi delayed equations including non-linear saturation gain as in[2]. In both cases, the delay line introduces an infinite dimensional phase space and the dynamics exhibited can be hyperchaotic (more than one positive Lyapunov exponent).

The number of positive Lyapunov exponents grows linearly with the delay time. This is a general characteristic of delayed systems. The Kaplan-Yorke dimension increases also linearly with the delay time. Therefore, very large dimensionalities can be achieved. However, the Lyapunov exponents that become positive as the delay time is increased have a very small absolute value. This, together with the fact that the largest positive Lyapunov exponents decrease as the delay time is increased, yields a saturation in the Kolmogorov-Sinai entropy. Although the system has a larger dimensionality when increasing the delay, its behaviour does not become more unpredictable. Consequently, for the purpose of using this chaotic output as a carrier for encoding a message, these results suggest that increasing the delay time beyond the value at which entropy saturates will neither yield a better masking nor improve the security of the chaotic encryption scheme.

In the electro-optical case, the feedback is nonlinear while the laser operates in the linear regime. The number of positive Lyapunov exponents as well as their value increases with the feedback in a linear way. Therefore the Kaplan-Yorke dimension and the Kolmogorov-Sinai entropy grow also linearly with the feedback strength (see Fig. 1).

In the all optical case, the feedback is linear and nonlinearites come from the laser itself. Keeping a constant pump value and increasing the feedback level, the number of positive Lyapunov exponents and their value increases up to a certain value of the feedback strength. Beyond this value, the largest Lyapunov exponent starts to decrease. For a slightly larger value, the second largest Lyapunov exponent also starts to decrease, and so on. As a consequence, the Kaplan-Yorke dimension does not grow linearly with the feedback strength any more and the Kolmogorov-Sinai entropy reaches a maximum and then decreases for larger feedback values. Keeping the feedback strength fixed and increasing the pump current, the Kolmogorov-Sinai entropy also goes through a maximum at an optimal pump value and may even fall to zero indicating a return to regular dynamics.

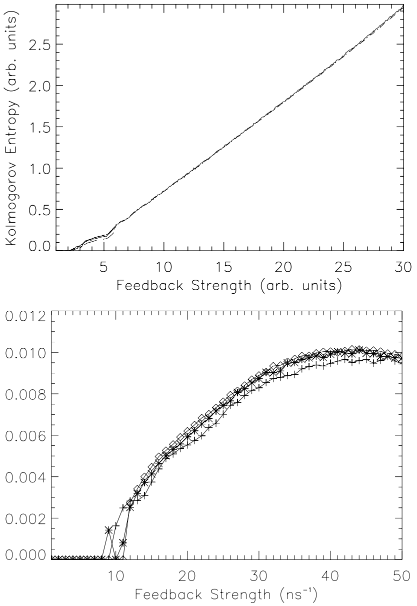

Figura 1. Kolmogorov-Sinai entropy as function of the feedback strength. Top: electro-optical feedback for delay times (in dimesionless units) 5, 10, 20, 50, 100, 250. Bottom: optical feedback for pump current 1.5 times threshold and delay times 200 ps (crosses), 300 ps (arterisks) and 1000 ps (squares).

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