Symmetric Brownian motor

A. Gomez-Marin* and J. M. Sancho**
Departament d’Estructura i Constituents de la Materia
Facultat de Física, Universitat de Barcelona
Av. Diagonal 647
08028 Barcelona

We all know that it is possible to extract some amount of mechanical work from a thermal bath at a temperature $T_2$ provided we have another bath at a lower temperature $T_1 < T_2$. Thermal engines are the devices that perform this task. All this is well known from elementary textbooks on thermodynamics. We also know from statistical mechanics that any object in a thermal bath exhibits random energy fluctuations of the order $k_B T$. These fluctuations are relatively very small for macroscopic objects but of very important relevance for nanometric objects such as biological motors: kinesins, dyneins, etc.1 We are also familiar with windmills, which are able to extract useful work from random winds by a proper adaptation to the wind direction. We can ask ourselves if it is possible to rectify thermal fluctuations by some appropriate mechanical devices. The engines which aim to get useful work by rectifying thermal fluctuations are called Brownian motors (BM). In fact, the paradigm of such speculations is Feynman’s famous ratchet and pawl machine2. During the last years, a lot of effort has been invested to study what has been called the ratchet effect. This is a mechanism that consists in breaking the spatial and temporal inversion symmetry of the system so that directed transport emerges, often enhanced by the thermal fluctuations3.

Here we present4 a model of a symmetric Brownian motor (SBM). See Fig.1. It consists on two ratchet potentials immersed respectively in two boxes at different temperatures $T_1$ and $T_2$. They are connected through a harmonic potential $V(\theta_1, \theta_2) = \frac{1}{2}k(\theta_1 - \theta_2 + \phi)^2$. We also add an external torque $\tau$ to get useful mechanical work. Note that the ratchets are placed as specular images. This is very important for the model. A key factor is that the harmonic interaction introduces a new control parameter, namely the phase shift $\phi$ of the coupling. We study the velocity, external work and efficiency of the SBM as a function of the temperatures of the baths and other relevant parameters. Moreover, the motor changes the sign of its velocity when the temperature gradient is inverted and it also shows a current reversal when the phase shift $\phi$ is varied.

Figura 1. Model for the symmetric Brownian motor (SBM).

We discuss generic properties of this type of motor and we show how to find an analytical prediction for the mean velocity as a function of $\phi$ that agrees very well with the numerical simulations performed. In Fig.2 we compare the predicted theoretical dependence of the velocity as a function of the phase $\phi$ to the values obtained by numerical simulations of the model. The analytical result traces very accurately the current inversion phenomenon. The fit is very encouraging and enlighting. The positions of the maxima are quite well determined, the inversion of current is clearly coincident with the simulations and even the little shift near $\phi = 0$ and $\phi = \frac{\pi}{2}$ is faithfully reproduced. This means that in our approximations we have kept the most essential ingredients. There is also an intuitive explanation of why the motor runs either forwards or backwards depending on $\phi$, which can be given by mapping our model into a multiplicative noise scenario or a temperature spatial dependence. A more extensive analysis can be found in4.

Figura 2. Comparison of the mean velocity $v$ versus the phase shift $\phi$ of the coupling obtained from numerical simulations (circles) and from analytical expression4 (solid line).

* agomez@ecm.ub.es
** jmsancho@ecm.ub.es