Collective effects of heterogeneity and stochasticity in interacting particle systems

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Systems studied traditionally in physics are made of identical (some times even indistinguishable) units (molecules, atoms, electrons). In recent years, methods and ideas developed in statistical physics have been applied to other disciplines (such as ecology, epidemic spreading, economy...), and systems outside the traditional realm of physics have been studied by physicists, focusing on collective and emergent phenomena in what is known as complexity science. This kind of systems are typically characterized by a large degree of heterogeneity among their units, and very often they can be modeled only at a stochastic level (since complete knowledge of all the variables, the precise dynamics of the units and the interaction with the environment is not available). One way to include the heterogeneity of the system is by considering that the interactions of the particles are not homogeneous but given by some connectivity network; an approach that has attracted enormous attention in the last years. An issue that has been less studied systematically is the role played by particle heterogeneity in the overall behavior of stochastic systems.

In this work, we show that the combined effect of stochasticity and heterogeneity can give rise to non-trivial results, and develop a general formulation to study systems of heterogeneous stochastic interacting particles.

For systems of 2-state independent particles, we show that the fluctuations of the collective variable decrease as the degree of heterogeneity is increased. Moreover, one can obtain precise information about the degree of heterogeneity in the system, by measuring only the collective variable (n) and its fluctuations:

$$\sigma_p^2 \equiv \langle p^2 \rangle - \langle p \rangle^2 = \frac{\langle n \rangle - \langle n \rangle^2/N - \sigma_n^2}{N}, \qquad (1)$$

with p_i some intrinsic parameter of the particle, that is distributed over the population. This expression is universal, regardless the specific form in which p_i is distributed, and an equivalent result is obtained for k-state systems for $k > 2$. Higher moments of the heterogeneity distribution are also related to higher moments of the collective variable.

We develop an approximated method to analytically study systems of heterogeneous interacting particles, and apply it to study the effect of heterogeneity in various models, such as the Kirman model[1].
We find that heterogeneity can amplify or reduce the fluctuations of the collective variable, depending on the way it is introduced and the particular form of the system.

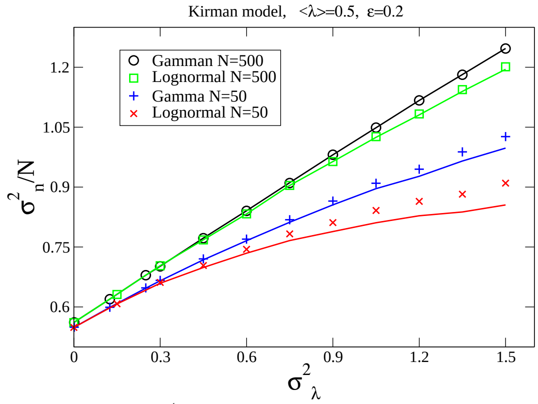

FIG. 1. Variance (rescaled by sytem size for better comparison) of the number of particles in state 1 as a function of the variance of the distribution of "influence" in Kirman model. Results coming from numerical simulations (symbols) and theoretical analysis (solid lines), for different system sizes and forms of the distribution.

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[1] A. Kirman, The Quarterly Journal of Economics, **108**, 137 (1993). This model was proposed to study herding behaviour in the context of stock markets and collective behavior in ant colonies. It corresponds to particles changing their states trough two mechanisms: spontaneous transitions at a rate ϵ, and induced transitions at a rate $\sum \frac{\lambda_j(1-\delta_{n_i,n_j})}{N}$, so that λ_j is the "influence" of agent j towards his current opinion.