A phase-field approach to actin-based motility of lamellar fragments

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Lamellar fragments are pieces of lamellipodia, the actin-based locomotion machinery of crawling cells. These fragments have been shown to exhibit spontaneous, sustained motion if properly deformed[1]. Despite its relative simplicity, a theoretical understanding of the minimal ingredients to explain this phenomenon is still lacking. A challenging question is how the treadmilling dynamics of actin, which polymerizes at the boundary, is coupled to the shape of the fragment to sustain motion, in particular in the absence of molecular motors. Recently, it has been shown that in an appropriate approximation, the flow of actin satisfies Darcy's law in an effectively two-dimensional geometry[2], thus reducing the dynamics to a free-boundary problem similar to that of viscous-fingering in Hele-Shaw cells, but with different boundary conditions[3].

Here we present a phase-field description of this free-boundary problem, as a tool to numerically integrate the fully nonlinear dynamics of this problem, aiming at a systematic study of the different families of steady propagating solutions, their stability and their basins of attraction. Such a diffuse-interface method is known to have important advantages with respect to sharp-interface methods (boundary-integral methods or confomal mapping techniques) in laplacian problems, in particular when interfaces adopt complex shapes. Most importantly, their advantage is most substantial when the viscosity of the displaced fluid is not neglected, a common approximation that may have to be relaxed in order to gain a more quantitative understanding of the problem and its biological relevance.

A phase-field strategy has already been used before within more complicated models[5,6]. The model we present concentrates on the effects of polymerization forces combined with friction on the substrate and with membrane tension. The absence of molecular motors in the description makes the model simpler and more amenable to theoretical discussion. Appropriate convergence tests have been performed showing that the model is quantitatively accurate and competitive with respect to other techniques. Our results in the fully nonlinear regime confirm that actin polymerization alone can sustain motion, and provide not only the families of stable steady shapes, but also a first characterization of their relative basins of attraction.

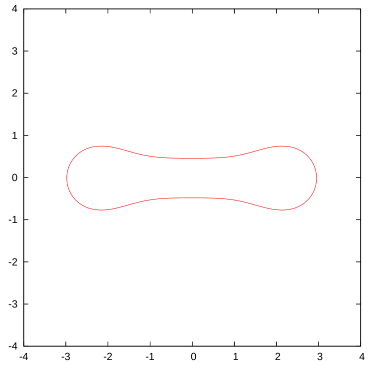

FIG. 1. Steady shape of a lamellar fragment obtained after the instability of a perturbed circle, using the phase-field model.

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