# Impact of roughness on the entropy of a granular gas in the homogeneous cooling state

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The dynamics of a dilute granular gas, modeled as a system of hard spheres colliding inelastically with constant coefficients of normal ($\alpha$) and tangential ($\beta$) restitution, can be described at a mesoscopic level by the (inelastic) Boltzmann equation for the one-body velocity distribution function $f(\mathbf{r}, \mathbf{v}, \boldsymbol{\omega}, t)$[1]. A granular gas is intrinsically out of equilibrium and thus, in contrast to the case of energy conservation (elastic collisions), the evolution of $f$ does not obey an H theorem, even if the system is isolated. On the other hand, it is worthwhile introducing the Boltzmann entropy density[2]

$$s(\mathbf{r}, t) = -\int d\mathbf{v} \int d\boldsymbol{\omega}\, f(\mathbf{r}, \mathbf{v}, \boldsymbol{\omega}, t) \ln f(\mathbf{r}, \mathbf{v}, \boldsymbol{\omega}, t).$$

Although $s(\mathbf{r}, t)$ does not qualify as a Lyapunov function in the inelastic case, its introduction is justified by information-theory arguments and also to make contact with the Boltzmann entropy density of a conventional gas. The local *relative* entropy per particle is

$$s^*(\mathbf{r}, t) = \frac{s(\mathbf{r}, t) - s_0(\mathbf{r}, t)}{n(\mathbf{r}, t)} = -\langle \ln R(\mathbf{r}, \mathbf{v}, \boldsymbol{\omega}, t)\rangle, \qquad (1)$$

where $s_0(\mathbf{r}, t)$ is the local equilibrium entropy density, $R(\mathbf{r}, \mathbf{v}, \boldsymbol{\omega}, t) \equiv f(\mathbf{r}, \mathbf{v}, \boldsymbol{\omega}, t)/f_0(\mathbf{r}, \mathbf{v}, \boldsymbol{\omega}, t)$, $f_0$ being the (two-temperature) local equilibrium distribution, and $\langle \cdots \rangle \equiv n^{-1} \int d\mathbf{v} \int d\boldsymbol{\omega} \cdots f$. Obviously, $s_0 \geq s$ and thus $s^* \leq 0$.

In general, the ratio $R$ can be expanded in a complete set of orthonormal polynomials $\{\Psi_{\mathsf{k}}\}$ as $R(\mathbf{r}, \mathbf{v}, \boldsymbol{\omega}, t) = 1 + \sum_{\mathsf{k}} A_{\mathsf{k}}(\mathbf{r}, t)\Psi_{\mathsf{k}}(\mathbf{r}, \mathbf{v}, \boldsymbol{\omega}, t)$, where $\mathsf{k}$ denotes the appropriate set of indices and $A_{\mathsf{k}}(\mathbf{r}, t) = \langle \Psi_{\mathsf{k}}(\mathbf{r}, \mathbf{v}, \boldsymbol{\omega}, t)\rangle$. In the *linear* approximation $\ln R \approx R - 1$, Eq. (1) becomes

$$s^*(\mathbf{r}, t) \approx -\sum_{\mathsf{k}} A_{\mathsf{k}}^2(\mathbf{r}, t).$$

Now we particularize to a homogeneous and isotropic system. In that case, neglecting coefficients $A_{\mathsf{k}}$ associated with polynomials of degree higher than four, we have

$$s^*(t) \approx -\frac{15}{8} a_{20}^2(t) - \frac{15}{8} a_{02}^2(t) - \frac{9}{4} a_{11}^2(t) - \frac{9}{4} b^2(t),$$

where the cumulants $a_{20}$, $a_{02}$, $a_{11}$, and $b$ are defined by

$$a_{20} = \frac{3}{5}\frac{\langle v^4\rangle}{\langle v^2\rangle^2} - 1, \quad a_{02} = \frac{3}{5}\frac{\langle \omega^4\rangle}{\langle \omega^2\rangle^2} - 1,$$

$$a_{11} = \frac{\langle v^2\omega^2\rangle}{\langle v^2\rangle\langle \omega^2\rangle} - 1, \quad b = \frac{9}{5}\frac{\langle (\mathbf{v}\cdot\boldsymbol{\omega})^2\rangle - \frac{1}{3}\langle v^2\omega^2\rangle}{\langle v^2\rangle\langle \omega^2\rangle}.$$

In this work we make use of a recent Sonine approximation for the collisional rates of change of $a_{ij}(t)$ and $b(t)$[3,4] to study the temporal evolution of $s^*(t)$ in the homogeneous cooling state. The asymptotic stationary value $s^*(\infty)$ is also analyzed as a function of both $\alpha$ and $\beta$.

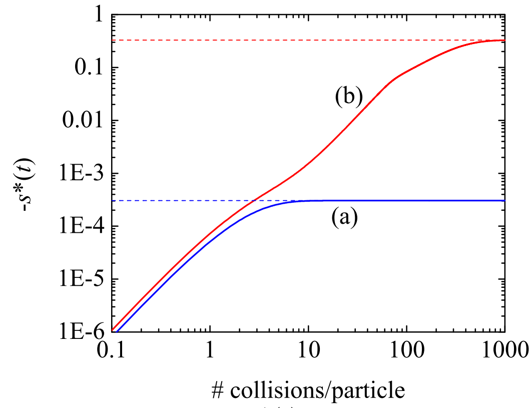


FIG. 1. Log-log plot of $-s^*(t)$ versus the number of collisions per particle for (a) $\alpha = 0.8$ and $\beta = -1$ and (b) $\alpha = 0.8$ and $\beta = -0.9$.

As an example, Fig. 1 shows the temporal evolution of the relative entropy $s^*(t)$ for (a) smooth particles ($\beta = -1$) and (b) slightly rough particles ($\beta = -0.9$), in both cases with $\alpha = 0.8$. We can observe the dramatic influence of roughness: both the relaxation time (measured by the number of collisions) and the stationary value $|s^*(\infty)|$ have increased by three orders of magnitude in the rough case (b) with respect to the smooth case (a).

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