Adler synchronization of spatial laser solitons pinned by defects

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Laser cavity solitons (LCS) are transverse, nonlinear, self-localized and dissipative states which have the potential for massive parallelism and the formation of complex arrays. Phase-locked bound states with solitons have been predicted in mode-locked lasers for the temporal case[1] and in lasers with saturable absorbers for the spatial case[2]. Corresponding phase-quadrature states have been observed in fiber lasers[3]. Here we present a different kind of soliton locking. We demonstrate experimentally and theoretically Adler-type locking and synchronization of spatial LCS in a vertical-cavity surface-emitting laser (VCSEL) with an external Bragg grating that provides frequency- selective feedback[4].

The dynamics of LCS in a semiconductor laser with feedback is well captured by a generic cubic complex Ginzburg-Landau equation coupled to a linear filter[6]:

$$\begin{aligned} \partial_t E &= g_0 E + g_2 |E|^2 E - i\partial_x^2 E + F + in(x)E, \\ \partial_t F &= -\lambda F + \sigma E \end{aligned} \qquad (1)$$

where $E(x)$ is the intra-cavity field, $F(x)$ the filtered feedback field, g_0 the linear gain and detuning, g_2 the nonlinear gain and dispersion, σ the feedback strength and λ the filter bandwidth. The reference frequency is set to the filter peak. $n(x)$ describes background defects that perturb the material refractive index.

For $n(x) = 0$, Eqs. 1 have exact solutions corresponding to stable single-frequency chirped-sech solitons[6] with two free parameters: location and phase. The interaction of two solitons makes them spiral slowly to fixed relative distances and phase differences around $\Phi = \pi/2$ unless merging takes place. $\Phi = 0, \pi$ are also possible but correspond to saddles that are either phase or distance unstable.

Small variations of $n(x)$ lead to pinning and small changes in the LCS frequency. If defects are located close enough, solitons interaction locks their frequencies to a common value. The phase difference Φ relaxes to stationary values that depend on the defect detuning $\Delta\omega = \omega_2 - \omega_1$ generated by $n(x)$. The dependence of Φ on $\Delta\omega$ for numerical simulations of (1) is shown in Fig. 1 (dots) for $|x_2 - x_1| = 1.5$ space units. Locking and synchronization occur only in the range $|\Delta\omega| < \Delta\omega_{th}$. Very similar results (triangles) have been obtained from numerical simulations of LCS in models of VCSELs with frequency-selective feedback that include the dynamics of the carriers[5]. The solid line refers to the results from the Adler model for synchronization between two coupled oscillators with different bare frequencies[7],

$$\dot{\Phi} = \Delta\omega - \varepsilon \sin(\Phi)\,. \qquad (2)$$

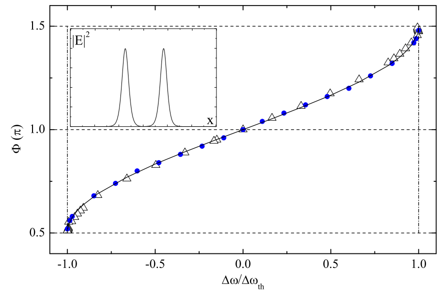

FIG. 1. Locked phase differences Φ of pinned LCS for different frequency detunings. The inset shows the spatial profile of the intensity.

The experiment was performed with a VCSEL and a volume Bragg grating (VBG) in a self-imaging configuration[6]. A piezo-electric transducer was used to minutely tilt the VBG with respect to the optical axis leading to a differential change in the feedback phase and allowing the tuning of $\Delta\omega$. When performing such a scan, a region of frequency and phase locking appears, identified in Fig. 2 by the region of high fringe visibility in the far field. As expected for the Adler scenario, in the locking region, the fringe phase varies smoothly and quasi-linearly with the detuning of the external cavity. The width of the locking range is close to the expected value of π and the transitions to and from frequency and phase-locking are rather abrupt

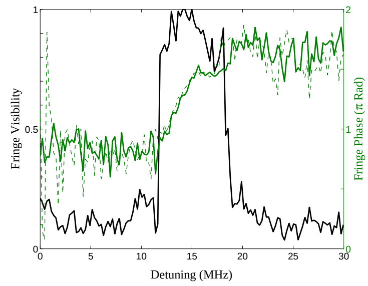

FIG. 2. Experimental fringe visibility (black) and fringe phase (green/gray)

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