Random walks on temporal networks

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Many natural and artificial networks evolve in time[1]. Nodes and connections appear and disappear at various timescales, and their dynamics has profound consequences for any processes in which they are involved. Until recently however, a large majority of studies about complex networks have focused on a static or aggregated representation, in which all the links that appeared at least once coexist. In recent years, the interest towards the temporal dimension of the network description has blossomed, and at the same time, researchers have started to study how the temporal evolution of the network substrate impacts the behavior of dynamical processes.

Here, we study how random walks, as paradigm of dynamical processes, unfold on temporally evolving networks[2]. To this aim, we use empirical dynamical networks of contacts between individuals in various social contexts, as recorded by the SocioPatterns project[3]. These dynamical networks exhibit heterogeneous and bursty behavior, indicated by the long tailed distributions for the lengths and strength of conversations, as well as for the gaps separating successive interactions. We underline the importance of considering not only the existence of time preserving paths between pairs of nodes, but also their temporal duration: shortest paths can take much longer than fastest paths, while fastest paths can correspond to many more hops than shortest paths. Interestingly, the appropriate rescaling of these quantities identifies universal behaviors shared across the datasets considered.

Given the finite life-time of each network, we consider as substrate for the random walk process the replicated sequences in which the same time series of contact patterns is indefinitely repeated. At the same time, we propose two different randomization procedures to investigate the effects of correlations in the real dataset. The "sequence randomization" destroys any temporal correlation by randomizing the time ordering of the sequence. This allows to write down exact mean-field equations for the random walker exploring these networks, which turn out to be substantially equivalent to the ones describing the exploration of the weighted projected network. The "statistically extended sequence", on the other hand, selects random conversations from the original sequence, thus preserving the statistical properties of the original time series, with the exception of the distribution of time gaps between consecutive conversations.

We perform numerical analysis both for the coverage and the mean first passage time properties of the random walker. In both cases we have found that the empirical sequences deviate systematically from the mean field prediction, inducing a slowing down of the network exploration and of the mean first passage time. Remarkably, the analysis of the randomized sequences has allowed us to point out that this is due *uniquely* to the temporal correlations between consecutive conversations present in the data, and *not* to the heterogeneity of their lengths.

Finally, we address the role of the finite size of the empirical networks, which turns out to prevent a full exploration of the random walker, though differences exist across the different cases. In this context, we have also shown that different starting nodes provide on average different coverages of the networks, at odds to what happens in static graphs. In the same way, the probability that the node i is reached by the random walker at any time in the contact sequence exhibits a common behavior across the different time series, but it is not described by the mean-field predictions for the aggregated network, which predict a faster process. Considering the fundamental prototypical role of the random walk process, we believe that these results could help to shed light on the behavior of more complex dynamics on temporally evolving networks.

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[3] `http://www.sociopatterns.org` .