

Detecting long-range teleconnections in the climate network via ordinal pattern time-series analysis

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Complex networks appear in almost all fields of science, examples being the internet, social interactions, food webs, biochemical reactions, brain functional networks, etc. For the purpose of modeling and forecasting, many systems lead naturally to the concept of networks of interacting elements, where one can define nodes and assign links among them depending on the (in principle, very complex) features of the system under study. It is by now well known that the Earth's climate is not only based on local factors, as the atmosphere connects far away regions through waves and advection of heat and momentum. This long-range coupling makes the network modeling approach of the Earth's climate extremely attractive and promising^{1,2}. By covering the Earth's surface with a regular grid of nodes, and by assigning links to connections between two different nodes via an analysis of their climate interdependency, the network approach has been shown to be able to extract novel and meaningful information.

Mutual information (MI) is a nonlinear measure, function of the probability density functions (PDF) that characterizes the time series in two different nodes $p_i(m)$ and $p_j(n)$, as well as of the joint probability function $p_{ij}(m, n)$. It can be defined as

$$M_{ij} = \sum_{m,n} p_{ij}(m, n) \log \frac{p_{ij}(m, n)}{p_i(m)p_j(n)}. \quad (1)$$

where the PDF needed for computation can be approximated by means of histograms. Besides that method, it has been proposed the use of ordinal patterns (OP). These are used in symbolic calculus and are first calculated from the time series by noting the value of a given point relative to its neighbors (see figure 1). Ordinal patterns do not constrain us to choose immediately adjacent points in order to make the patterns. We can construct them letting a time interval between points, and in this way we can consider different time scales.

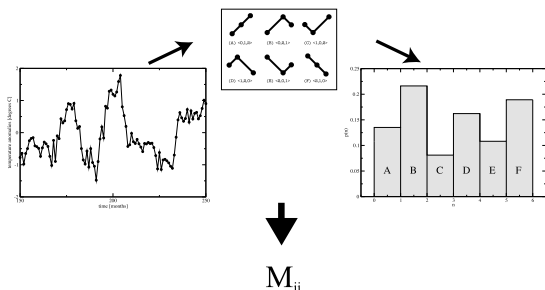


FIG. 1. We transform the original time series in a sequence of OPs (shown for $n=3$), then compute the PDFs of the OPs and finally compute the Mutual Information using Eq. 1.

We analyze monthly averaged surface air temperature anomalies (SAT field) from the NCEP/NCAR reanalysis¹.

We have calculated M_{ij} from the PDFs of the OPs constructed from consecutive months and from consecutive years. We have used OPs of length 4 and thus there are $4! = 24$ different possible OPs. Afterwards we construct the elements A_{ij} of the network with the elements of M_{ij} over a threshold of significance (calculated using surrogate data). In figure 2 we graphically represent our networks showing the Area Weighted Connectivity: $AWC_i = \sum_j^N A_{ij} \cos(\lambda_i) / \sum_j^N \cos(\lambda_j)$ over the world map.

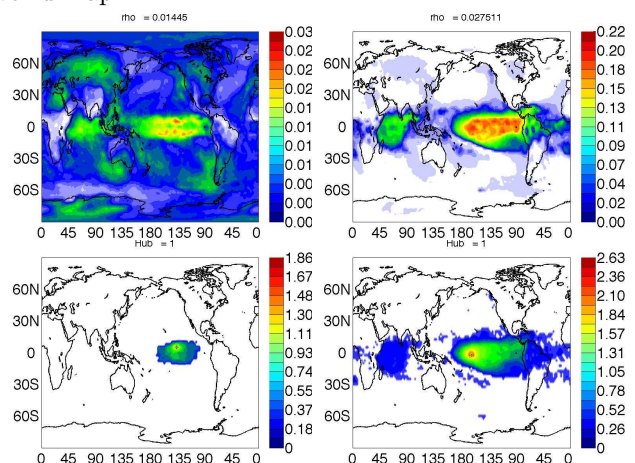


FIG. 2. (first row) AWC maps calculated from OPs constructed with adjacent months (left) and OPs constructed with adjacent years (right). On the second row we show the corresponding most connected node (hub) in the network: the color code indicates the mutual information of the hub with the other nodes. We observe tele-connections in the inter-annual time scale (left) but not in the intra-season time scale (right).

We have shown that these techniques are useful for constructing climate networks on different time scales. This provides a novel approach to climate network analysis that can be extended to other, shorter or longer time-scales, such as those in meteorology or in paleoclimatology.

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¹ M. Barreiro, A. C. Marti y C. Masoller. *Chaos*, **21**:013101(2011).

² J. F. Donges, Y. Zou, N. Marwan and J. Kurths. *Eur. Phys. J. Spec. Top.* **174**:157 (2009).