Synchronization in delayed mutually coupled optoelectronic oscillators

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In this work we study the synchronization between two delayed mutually coupled optoelectronic oscillators with intrinsic delay. In particular we consider the interplay of the different delays in achieving synchronized behavior. We analyze the stability of the synchronized solutions computing the spectrum of Lyapunov exponents.

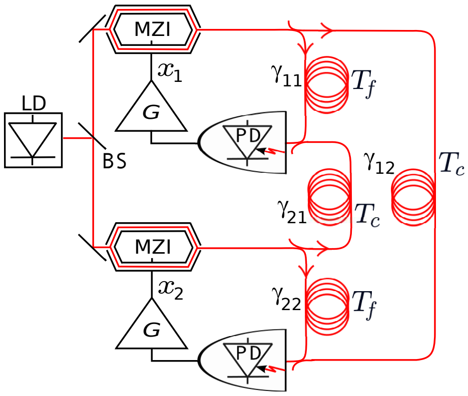

FIG. 1. Setup.

The setup is based on two electro-optical delay systems mutually coupled as shown in FIG. 1. Light from a cw semiconductor laser (LD) is splitted into two beams, each of which enters a system[1] with a Mach-Zender Interferometer (MZI). A fraction of light γ_{ii} enters the electro-optical loop of oscillator i with self-feedback delay $T_{ii} = T_f$, while a fraction $\gamma_{ij,j\neq i}$ is coupled to oscillator j with coupling delay $T_{ij,j\neq i} = T_c$. Light is detected by a photo-diode (PD) and the electrical signal goes through a band-pass amplifier of gain G_i and low and high cut-off characteristic times $\theta_i = 5\mu s$ and $\tau_i = 25ps$, respectively. The dynamics of the electrical signal x_i is:

$$x_i(t) + \tau_i \frac{dx_i}{dt}(t) + \frac{1}{\theta_i}\int_{t_0}^{t} x_i(s)ds = \beta C_i,$$
$$C_i = \gamma_{ii}^2 \cos^2(z_{ii}) + \gamma_{ji}^2 \cos^2(z_{ji}) +$$
$$+ 2\gamma_{ii}\gamma_{ji}\cos(z_{ii})\cos(z_{ji})\cos(z_{ii} - z_{ji}),$$

where $i, j = 1, 2$, $z_{ji} = x_j(t - T_{ji}) + \phi_j$, ϕ_i is an offset phase and β is proportional to the pump power.

Here we keep fixed $\gamma_{11} = \gamma_{12} = 0.5$ and $\phi_i = 0.25\pi$. For very low values of γ_{22} and γ_{21} (a configuration similar to unidirectional coupling) increasing β we find steady states, periodic solutions of period $2T_f$ and chaotic solutions. For intermediate values of γ_{22} or γ_{21}, the $2T_f$ periodic solutions become unstable, but we find other periodic solutions in certain cases. More precisely, when T_c and T_f satisfy the ratio $T_c/T_f = (2m + 2)/(2m + 1)$ for any integer $m \geq 0$, there are multiple stable periodic solutions with period $T_m = 2T_l/(2m + 1)$, being $T_l = T_c - T_f$. In these solutions, x_1 and x_2 are anitisynchronized with zero lag: $x_2(t) = -x_1(t)$. An example is shown in FIG.2, where almost square waveforms arise for $\tau_i << T_c = 40ns$, $T_f = 30ns << \theta_i$. From the Lyapunov spectrum shown in FIG.3, one can identify a region in the parameter space of $\gamma_{22} = \gamma_{21}$ where two exponents are zero and the rest are negative; this reveals that this solutions exhibit stability and double periodicity. Further increasing the pump or the coupling, periodic solutions become unstable and the system gets chaotic.

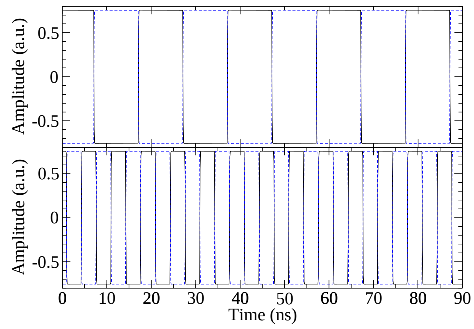

FIG. 2. Stable periodic solutions synchronized in antiphase with fundamental frequency (top panel, $m = 0$) and first harmonic (lower panel, $m = 1$). We have taken $\beta = 5$ and $\gamma_{22} = \gamma_{21} = 0.05$.

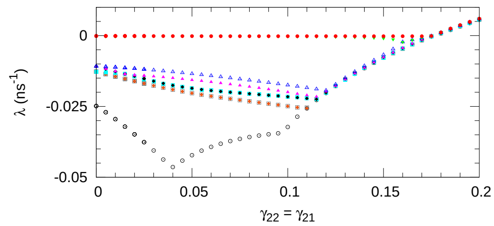

FIG. 3. First nine Lyapunov exponents of trajectories with periodicity shown in top panel in FIG.2, $\beta = 5$ and coupling $\gamma_{22} = \gamma_{21}$. Stability is lost around $\gamma_{22} = \gamma_{21} = 0.175$, where one Lyapunov exponent becomes positive.

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[1] J. P. Goedgebuer, P. Levy, L. Larger, C.-C. Chen, and W.T. Rhodes, IEEE J. Quantum Electron. **38**, 1178 (2002).

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