Role of delay in the stochastic birth and death process.

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Fluctuations play an important role in many areas of science, and their study has become a well defined discipline. Delay in the interactions is also a common phenomenon in natural and artificial systems, and it is well known that it can alter qualitatively the dynamical behavior, for example inducing oscillations or even chaos.

The combined effect of stochasticity and delay is not completely understood. Stochastic processes that include delay are analytically difficult due to their non-Markovian character. Most theoretical studies consider a Langevin approach (stochastic differential equations) or systems in discrete time (where delay can be accounted for by increasing the number of variables). Stochastic models with continuous time but discrete variables are the natural description of many systems such as chemical reactions, population dynamics, epidemics, etc. In some cases, the discreteness can be a major source of fluctuations.

In this work we develop a rigorous derivation of the stochastic description of birth and death processes that include delay. The processes are schematically described by:

$$\emptyset \xrightarrow{C(n)} X, \qquad X \xrightarrow{\gamma} \emptyset, \tag{1}$$

with n the number of species X. Here either the creation or the degradation reactions (that are initiated at a rate $C(n)$ and γ respectively) take a time τ to be completed.

For the case of delay in the degradation[1], we solve the process exactly and find that the exact solution for the probabilities leads to equations for the mean values that do not comply with simple intuitive arguments and that oscillatory behavior does not exist (while it is usually believed to be present in this type of system). This clarifies and warns about the derivation of dynamical equations describing the evolution of the concentrations in cases in which delay plays a role. The exact solution is specially valuable for small system sizes, where approximated schemes typically fail.

For the case of delay in the creation[2] and feedback we develop an approximated analytical treatment that allows us to study the effect of delay and show that the delay can alter qualitatively the character of the fluctuations, amplifying them in the negative feedback case and reducing them in the positive feedback case. We also consider the situation with distributed delay and show that the effect of the delay decreases as the delay distribution becomes wider.

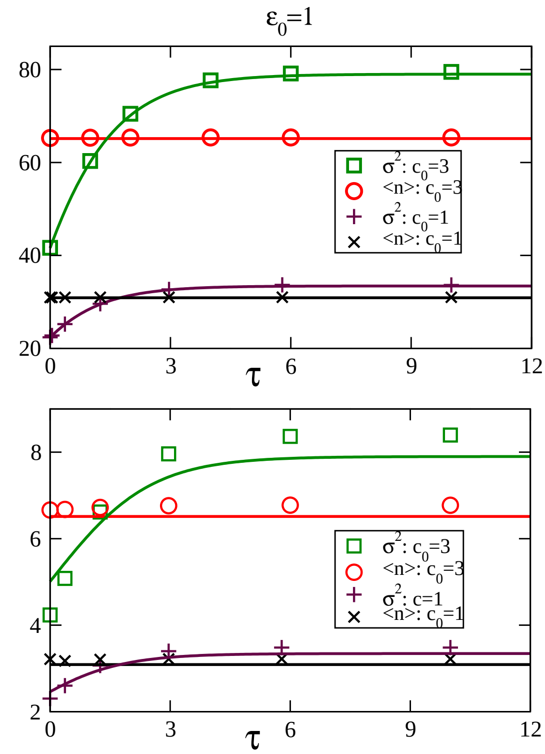

FIG. 1. Steady state average $\langle n \rangle_{st}$ (dashed lines) and variance σ^2_{st} (solid lines), for the delay in the creation with negative feedback, as a function of the delay time τ, for a creation rate $C(n) = \frac{c_0 \Omega}{1+\frac{\epsilon_0}{\Omega} n}$ with $c_0 = 3$ (upper part of each panel) and $c_0 = 1$ (lower part of each panel), and two system sizes (Ω) (upper and lower panel) and $\epsilon_0 = 1$ in both cases. In each case, we plot with symbols the results coming from numerical simulations and by lines the theoretical expressions. Note that the fluctuations change from sub-Poissonian to super-Poissonian as the delay is increased.

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[1] L. F. Lafuerza and R. Toral, Phys. Rev. E 84, 051121 (2011).

[2] L. F. Lafuerza and R. Toral, Phys. Rev. E 84, 021128 (2011).