Towards a statistical mechanics of neurons: The excitable-wave mean-field approximation

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Most neurons present cellular tree-like extensions known as dendrites, which receive input signals from synapses with other cells. Some neurons have impressive dendritic arbors [as illustrated in Fig. 1 (a)], and exhibit dendritic excitability through the expression of a variety of voltage-gated ion channels. Why neurons have such elaborated spiking dendrites?

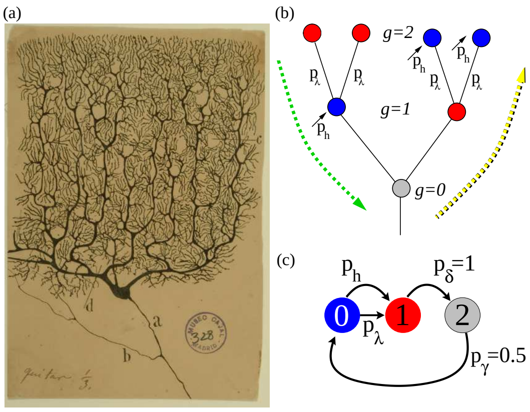

FIG. 1. Model of an active dendritic tree. (a) A famous drawing by Ramon y Cajal of a human Purkinje cell. (b) Excitable elements (circles) connected (bars) in a Cayley tree topology with G = 2 layers and coordination number z = 3 (one mother and two daughter branches). Dendritic branchlets are driven by independent Poisson stimuli (small arrows). (c) Each dendritic branchlet can be in one of three states: quiescent (0), active (1) or refractory (2).

Dendrites have been studied for decades, but the field of *dendritic computation* is still in its infancy. Since the dendritic arbor and its synapses are always suffering spatio-temporal restructuring, one major theoretical goal is to propose reliable computations that do not depend on an exact tree morphology, static synaptic density or fine details of dendritic branchlets. A robust and recently proposed function for active dendritic trees is to enhance the neuronal dynamic range[1], which measures the range of incoming stimulus intensity giving rise to distinguishable neuronal response. However, most of the insights for such proposal were gained based on numerical simulations and a few experimental data. A formal mathematical attempt to tackle the problem was still missing.

We analytically study the input-output properties of a neuron whose active dendritic tree, modeled as a Cayley tree of excitable elements, is subjected to Poisson stimulus, see Fig. 1 (b) and (c). Traditional mean-field approaches, single-site (1S) and two-site (2S) mean-field approximations, incorrectly predict a phase transition which is not allowed in the model. We introduce a new excitable-wave (EW) mean-field approximation, which keeps track of the direction of propagation of the excitable waves, thus reproducing the main results of the model[2]. As shown in Fig. 2, the novel EW mean-field approximation correctly describes the dynamic range (Δ) of the dendritic arbor for any probability of propagation of excitation from branchlet to branchlet (p_λ).

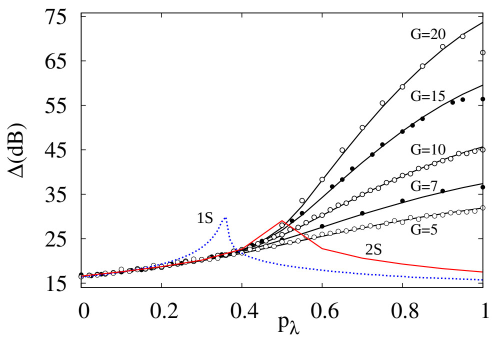

FIG. 2. Dynamic range as a function of the coupling parameter for 1S (G = 10), 2S (infinite tree), EW (black lines) mean-field approximations compared to simulations (open and closed symbols) for different trees sizes G.

Based in properties of the excitable media, the active dendritic trees analogically perform non-linear signal transformations that generate robust and efficient computations. Both simulations and analytical treatments of excitable trees enable the test of several hypothesis and the comparison with experiments in the area of dendritic computation.

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[2] L. L. Gollo, O. Kinouchi, and M. Copelli, Phys. Rev. E 85, 011911 (2012).