Patterns and Domain Walls in Generalized Swift-Hohenberg Dynamics

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Near a pitchfork bifurcation, where a uniform steady state becomes unstable towards two branches of equivalent uniform steady states, the dynamics may be reduced to the following order parameterlike dynamics, for two-dimensional systems with inversion and space-inversion symmetry:

$$\partial_t \sigma = [\epsilon + \mu \nabla^2 - \nabla^4]\sigma - \sigma^3 \tag{1}$$

where $\sigma(x, y, t)$ is the order parameterlike variable and $\nabla^2 = \partial_x^2 + \partial_y^2$. Fourth derivatives are relevant for $\xi_0^2 << 1$. The trivial state $\sigma = 0$ is stable for $\epsilon < 0$ and unstable for $\epsilon > 0$. For $\epsilon > 0$, this equation admits two branches of equivalent stable uniform steady states given by $\sigma_0(\pm) = \pm\sqrt{\epsilon}$. In some cases, though, nonlinearities may also depend on space derivatives of the order parameterlike variable. This is the case, for instance, in sodium vapor cells, where the two nontrivial equivalent solutions may become modulationally unstable[1]. A systematic expansion in powers of these derivatives gives, at lowest orders, in the case of space-inversion symmetry:

$$\partial_t \sigma = [\epsilon + \mu \nabla^2 - \nabla^4]\sigma - \sigma^3 - \gamma\sigma^2\nabla^2\sigma - \kappa\sigma(\vec{\nabla}\sigma)^2 \tag{2}$$

This dynamics admits the same steady states than (1), but the stability of the bifurcating states is different. In the case of the dynamics observe in optical systems[1], the bifurcation parameter also determines the strength of the nonlinearities, and the dynamics can be qualitatively described by the following model

$$\partial_t \sigma = -[\lambda_c - \mu\nabla^2 + \nabla^4]\sigma + \lambda[\sigma - \sigma^3 - \gamma\sigma^2\nabla^2\sigma - \kappa\sigma(\nabla^2\sigma)^2] \tag{3}$$

where λ is the order parameter. For $\lambda < \lambda_c$, the trivial steady state $\sigma = 0$ is stable. For $\lambda > \lambda_c$, it becomes unstable, via a pitchfork bifurcation, versus the two equivalent uniform steady states $\sigma_0(\pm) = \pm\sqrt{\frac{\lambda-\lambda_c}{\lambda}}$ which tend to 1 for increasing λ. One also finds that uniform bifurcating states become unstable at

$$\lambda_p = \lambda_c + (\frac{\sqrt{2} + \sqrt{2 + \gamma\mu}}{\gamma})^2 \tag{4}$$

versus spatial modulations of wavenumber

$$|q_p| = \sqrt{\frac{\gamma(\lambda_p - \lambda_c) - \mu}{2}} \tag{5}$$

leading to hexagonal patterns around the two $\sigma_0(\pm)$ solutions.

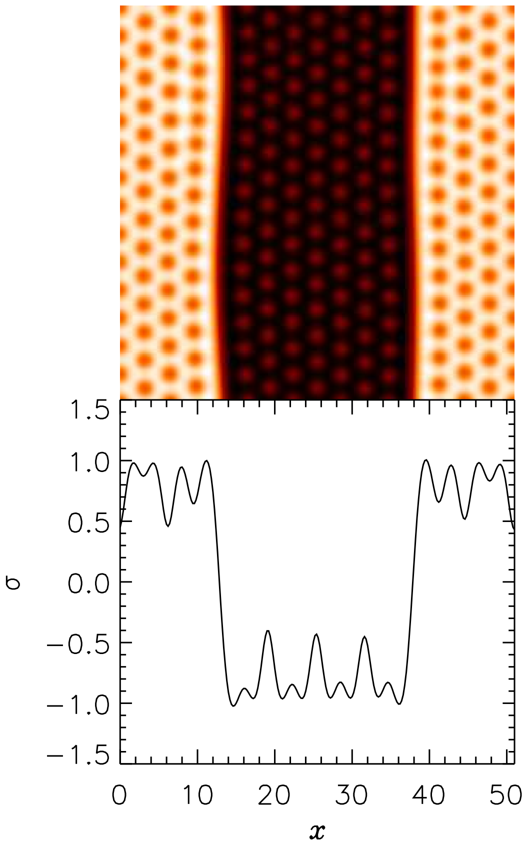

FIG. 1. Snapshot of the evolution of a two opposite domain walls between hexagonal patterns around the $\sigma_0(\pm)$ solutions. The bottom panel shows a transverse cut of the pattern shown at the top.

This leads to very complex dynamics including domain walls between patterned solutions (see Fig. 1). In this work we explore the dynamics of such general model and compare the results with experimental observations in an optical system.

[1] A.Aumann, E.Grosse Westhoff, R.Herrero, T.Ackermann and W.Lange, J.Opt.B: Quantum Semiclass. Opt. **1** 166 (1999); E.Grosse Westhoff, V.Kneisel, Yu. A.Logvin, T.Ackermann and W.Lange, J.Opt.B: Quantum Semiclass. Opt. **2** 386 (2000); M. Pesch, W. Lange, D. Gomila, T. Ackemann, W.J. Firth, and G.-L. Oppo, Phys. Rev. Lett. **99**, 153902 (2007).