On the irrationality of consensus in heterogeneous networks

Daniele Vilone[*,1], José J. Ramasco[1], Maxi San Miguel[1], Angel Sánchez[2,3]
[1]*Instituto de Física Interdisciplinar y Sistemas Complejos, CSIC-Universidad de las Islas Baleares, 07122-Palma (Mallorca)*
[2]*Grupo Interdisciplinar de Sistemas Complejos, Departamento de Matemáticas, Universidad Carlos III de Madrid, 28911-Leganés (Madrid)*
[3]*Instituto de Biocomputación y Física de Sistemas Complejos, Universidad de Zaragoza, Campus Río Ebro, 50018-Zaragoza*

The problem of reaching consensus in social systems is a very interesting issue[1]. Actually, the appearance of uniform behaviours, such as the convergence to the same opinion, can be observed in different situations. Understanding when and how this phenomenon occurs is one of the main goals of sociophysics. In this work we present a new model where agents chamge their opinion, which can assume one of two possible states, say A and B, by means of a mixed dynamics. At each elementary time step, we pick up an individual at random. This individual evolves following the voter model (VM) dynamics[1] with probability q, and with probability $1-q$ according to the coordination game (CG) dynamics with imitate-the-best update rule[2]. In practice, the chosen agent imitates with probability q a randomly chosen neighbour, and with probability $1-q$ the one which collects the largest total payoff in a round of the game (of course, if no neighbour performs better than the chosen agent, nothing happens). The CG has the simplest possible rule: an agent earns 1 for each neighbour with her same opinion, 0 otherwise. We can say, from a sociological point of view, that the VM dynamics is an irrational way to evolve, whilst the CG rule represents a rational behaviour.

The evolution and the final fate of such a system strongly depend on the value of the parameter q and on the topology on which it runs. The common feature of all the results we collected in many numerical simulations is the following: if the system reaches the consensus in one limit ($q = 0$ or $q = 1$) remaining disordered in the other one, we observe a smooth cross-over between the two limits. This is the case in one- and two-dimensional lattices: the system ends up reaching total consensus for $q = 1$ and in a frozen but disordered configuration for $q = 0$, while in a complete graph (mean-field) in the thermodynamical limit it reaches the consensus for $q = 0$ and remains disordered (in an active state) for $q = 1$.

The most interesting phenomenology appears on random topologies. Let us consider for instance what happens on an Erdös-Rényi (ER) network. In this case, neither a pure VM dynamics nor a pure CG one drive the system to the complete ordering (in the thermodynamical limit). In particular, for $q = 0$ it reaches a frozen disordered configuration, while for $q = 1$ it ends in an active state, in both cases with individuals of opposite opinion still coexisting. Conversely, if $0 < q < 1$, after a transient (whose length depends on q) the system goes always to consensus. The time τ_q needed to reach the final ordered state is not a monotonic function of q, and there is an optimum value q^* where it is minimum, as shown in Fig. 1, that also proves how these results are not consequence of finite size effects. It is quite noticeable that even a very small mixture of the two dynamics is enough to make the system reach consensus, as if each dynamics works as a noise with respect to the other, and as if the disordered state were an unstable configuration. Moreover, the ordering time diverges differently in the two limits:

$$\tau_{q\to 0^+} \sim K^{\frac{1}{\nu q}} \quad , \qquad\qquad \tau_{q\to 1^-} \sim \frac{1}{1-q}$$

where $K > 1$ and ν are suitable constants. This peculiar behaviour is similar to what is observed in some glassy transitions[3].

Interestingly, this picture is qualitatively the same also with other kinds of complex networks, as for instance small-world and scale-free networks. We also provide a theoretical analysis and a sociological interpretation of the results: in this respect, our main conclusion is that global consensus on an issue requires some people making their decision in a non-strategic manner.

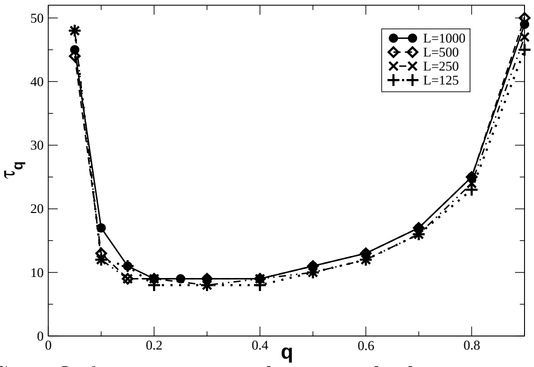

FIG. 1. Ordering time as a function of q for a system in an ER network (average degree $\langle k\rangle = 14$) and different sizes L.

* daniele.vilone@gmail.com

[1] C. Castellano, S. Fortunato and V. Loreto *Rev. Mod. Phys,*, **81**, 591 (2009).
[2] C.P. Roca, J.A. Cuesta and A. Sánchez, *Phys. of Life Rev.*, **6**, 208 (2009).
[3] J.P. Bouchaud, L.F. Cugliandolo, J. Kurchan amd M. Mézard in "Spin-glasses and random fields", Ed. A.P. Young, World Scientific (1998).