Synchronization transitions in a growing complex network of Stuart-Landau oscillators

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Complex Systems composed out of dynamical elements exhibit universal collective behaviors in fields like Physics, Biophysics, Neuroscience, Social Sciences, etc., and the interplay between local dynamics and topology of interaction can be often rationalized[1–3]. In the present work we are interested in transitions from an incoherent or quiescent state to a coherent state occuring as a function of the number of interacting oscillators (depending also on the local dynamics and coupling topology). Two different transitions of this type, that have been recently reported in the literature, are known as Crowd Synchrony (CS) and Dynamical Quorum Sensing (DQS) transitions. The first is characterized by a smooth transition first observed in London Millenium Bridge[4]. The second is a sharp transition introduced to describe the sudden increase of production of signaling molecules in bacteria colonies and that was also observed in yeast cells[5] and coupled chemical oscillators[6].

In Ref.[7] it was recently reported that a system of delay coupled lasers with a star network topology may exhibit both transitions when the number of coupled lasers exceeds a certain threshold number. This study showed that the type of transition that is found, CS or DQS, depends on the existence (or lack) of oscillations in the uncoupled lasers, depending on whether they are above (below) threshold, respectively. More recent investigations have revealed that the coupling strength and the number of lasers play an equivalent role in the transition, and demonstrated the existence of a second transition, past the DQS, to a more synchronized state[8].

However, it is not well understood which are the minimal characteristics of a system, both regarding local dynamics and coupling topology, that allow it to show one, the other or both types of transitions.

So, in the present work we introduce and study a minimum model that contains what we think are the essential ingredients to reproduce the DQS and CS transitions. The system is constituted of N complex Stuart-Landau oscillators, z_j, coupled through a common damped linear oscillator, F:

$$\dot{z_j} = (\mu_j + i\theta_j)z_j - |z_j|^2 z_j + \kappa(F - z_j), \qquad (1)$$

$$\dot{F} = (-\Omega + i\Delta)F + \kappa \sum_{j=1}^{N} z_j, \qquad (2)$$

where μ is the gain factor, θ is the detuning frequency and κ is the coupling strength. Ω is the filter bandwidth and Δ the central frequency.

As the DQS can appear when the oscillators are identical, when they exhibit a sudden transition to synchronization as a function of N, we have described the problem using the Master Stability Function (MSF) technique[9]. Instead, the CS transition appears to require heterogeneity in the interacting elements. So, we have considered the extension of the MSF formalism that allows for some heterogeneity among the oscillators[10]. In addition, we have used the Ott-Antonsen technique to analyze the CS synchronization transition[11]. In this case, the role of a passive filtering of a finite number of frequencies is discussed. Finally, the effect of time delay in the coupling channel is also considered and compared with the case of instantaneous coupling.

Our study suggests that CS and DQS could be observed in a large variety of natural systems and opens the door to the possibility of investigate these transitions in other coupling topologies.

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