Microscopic tip fluctuations drive the morphology of macroscopic Fisher fronts

Svetozar Nesic[1], Rodolfo Cuerno[1], Esteban Moro[1,2,3]

[1] *Departamento de Matemáticas & GISC, Universidad Carlos III de Madrid, 28911 Leganés, Spain*

[2] *Instituto de Ciencias Matemáticas CSIC-UAM-UCM-UC3M*

[3] *Instituto de Ingeniería del Conocimiento, Universidad Autónoma de Madrid, 28049 Madrid, Spain*

We have investigated the dynamics of the morphology of Fisher waves subjected to small intrinsic noise, as described by the Fisher-Kolmogorov-Petrovsky-Piscounov (FKPP) equation,[1]

$$\frac{\partial \rho}{\partial t} = D\Delta\rho + \rho - \rho^2 + \sqrt{\rho/N}\ \eta(x,t), \qquad (1)$$

where $\eta(x,t)$ is a Gaussian white noise and N is approximately the number of particles per unit volume.[2] For $N = N_c$, equation (1) undergoes a phase transition between an active phase ($\rho \neq 0$) and an absorbing state ($\rho = 0$).[4–6] In addition, for very large $N \gg N_c$ the FKPP equation displays *pulled fronts* in which the active phase invades the absorbing state.[3] Although intrinsic noise is really small for large N, it produces strong corrections to the velocity of the front when compared to the deterministic $N \to \infty$ equation.[8]

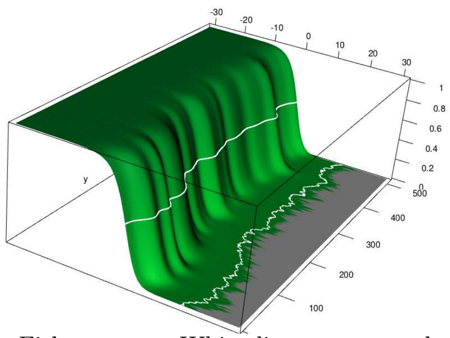

FIG. 1. Fisher waves: White lines correspond to equipotential lines at $\rho = 1/2$ (front) and $\rho = 1/N$ (cut-off). The light gray color represents the area where $\rho = 0$.

In this communication we study whether those strong corrections also happen in the kinetic roughening of the front. Using a non-negativity preserving algorithm to integrate equation (1),[5,6] we have found that the large-scale fluctuations in the morphology of the front line, see Fig. 1, belong to the 1D Kardar-Parisi-Zhang (KPZ) universality class.[7] As in the zero-dimensional case,[9] we find that the dynamics of the front in the cut-off microscopic line where $\rho \simeq 1/N$ (see Fig. 1) drive the dynamics of the macroscopic system (see Fig. 2). On the left panel of this figure, we show the time evolution of the roughness for both the front and the cut-off lines. Although the small-scale behavior is strictly different (see zoom in inset), the large-scale properties are indistinguishable. Hence, the 1D KPZ asymptotic behavior of the cut-off line is inherited by the front line, see right panel in the same figure, where the power spectral density function (PSD) is shown for both lines at long times. Notice that the strong short-scale fluctuations in the cut-off line are suppressed for the front, that looks much smoother at such scales, see Fig. 1. Morever we have also found that the 1D dynamics of the cut-off line propagates back to the front line (see inset of Fig. 2) and that it happens with a time ag that depends logarithmically on N.

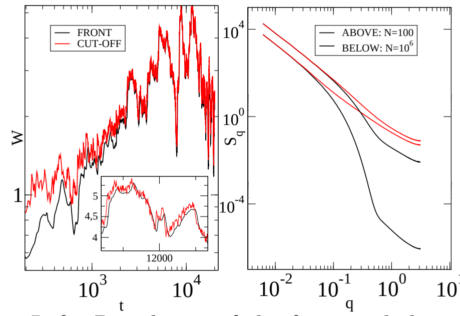

FIG. 2. Left: Roughness of the front and the cut-off lines vs time (single realiaztions). Inset: Zoom of main panel. Right: PSD functions of front (lower curves) and cut-off (upper curves) lines vs wave-vector q for times $t > 10^4$, for two values of N. For small q (long-rage correlations), the front and cut-off display the same behavior, which is not the case for small q.

We have thus found that the large-scale dynamics of the macroscopic front is determined by the evolution of the region that is driven by microscopic fluctuations, which is an unusual effect in front dynamics and creates macroscopic observable effects from microscopic noise.

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