The role of noise and initial conditions in the asymptotic solution of a continuous opinion dynamics model

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The models of "opinion dynamics" focus on the processes of opinion formation within a society consisting of an ensemble of interacting individuals with diverse opinions. A wide variety of models, mostly inspired by statistical mechanics and nonlinear physics, has been developed in order to deal with the different phenomena observed in real societies[1]: emergence of fads, minority opinion survival and spreading, collective decision making, emergence of extremism and so on. One of the main problems addressed by some of these models is whether the opinion formation processes within a society will eventually lead to the emergence of a consensus, with a vast majority of the agents adopting a similar opinion, or to the fragmentation of its constituent individuals into different opinion groups.

A model was introduced in 2000 by Deffuant, Weisbush and others[2], in the context of a European Union project for the improvement of agri-environmental policies, in order to describe the dynamics of continuous opinions under bounded confidence. In this model, each agent is therefore characterized by an opinion which is a continuous variable taken from a given interval. In its original version, these agents meet in random pairwise encounters and, if the difference between their continuous-value opinions is less than a certain confidence level, they interact, that is, they adjust their opinion towards the opinion of the other agent.

While most of the existing literature has focused on the important role played by the bound of confidence parameter, a topic that has received much less attention is the dependence of the model on the initial conditions, that is, on the initial distribution of opinions among the agents. As a first and naïve argument, we could notice that the model, as has been presented, conserves the mass and the mean opinion of the population, so the position of the final clusters will depend on the mean of the initial distribution. However, also different initial conditions with the same mean opinion could give rise to different final configurations. In fact, any combination of delta-functions is a steady state solution of the master equation describing the model, provided these delta-peaks are separated by more than a distance ϵ and they conform to the mean opinion conservation[4]. This has been the object of our study, and we have shown that it is perfectly possible to force or avoid a consensus by varying the initial distribution of opinions. Thus, we have sketched the structure of attractors of the dynamical system, by means of the numerical computation of the time evolution of the agents density. In particular, we have used a discrete density-based reformulation of the model as discussed by J. Lorenz[3].

In order to avoid having perfect consensus within each cluster, i.e., all the participant agents sharing exactly the same opinion, Toral, Pineda and Hernández-García have recently presented an extension of the original Deffuant-Weisbush model taking into account some additional randomness[4]. Thus, a noise is introduced in the dynamics as a *free will*, which allows for the agents to change their opinion from time to time to a randomly chosen value. For a uniform opinion distribution of the noise, this is equivalent to a quenched noise —as contrary to some other more adaptive noise approaches[5]—, that is, to allowing each agent to go back to a certain preferred opinion from time to time. As a result of this extension, there exists a transition between an organized and a disoganized phase at a critical value of the noise intensity. In the disordered state, corresponding to quite high noise rates, there is no cluster formation, as noise is stronger than nucleation processes; on the other hand, in the ordered state, corresponding to lower noise rates, we can still clearly observe the formation of clusters, even if they broaden with respect to the noiseless case. An interesting feature of this extension is that the average opinion of the system tends to the average of the noise opinion distribution, independent of that of the initial condition. Therefore we have also studied the influence of the initial distribution of opinions in this noisy case. There, we have shown that, even if the importance of the initial distribution regarding the average opinion is replaced by that of the noise distribution, the former has still some influence in determining the bifurcation patterns.

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