Shape and motility of actin lamellar fragments

Carles Blanch-Mercader and Jaume Casademunt*
Departament Estructura i Constituents de la Matèria, Universitat de Barcelona, Av. Diagonal 647, Barcelona, Spain

Lamellar fragments of keratocytes are pieces of the actin-based motile machinery extracted from those cells. These fragments lack nucleus, microtubules and most organelles, but retain the minimal ingredients to generate motion. Experimental observations[1] show that such fragments are capable to generate and sustain spontaneous directional motion if the circular symmetry is externally or spontaneously broken. A theoretical understanding of this phenomenon is still lacking. In particular, an interesting open question is whether actin polymerization forces plus friction, without the action of molecular motors, are sufficient to sustain motion.

We base our study on a physical model that was recently introduced by Callan-Jones et al[2]. It assumes a polar nematic continuous description[3] of the gel of actin and assumes that the dynamics of the polymerization can be slaved to the slow membrane dynamics. Assuming that actin treadmilling is controlled by polymerization at the membrane with velocity v_p and uniform depolymeriztion in the bulk, the velocity field can be shown to satisfy Darcy's law and therefore be reduced to a laplacian pressure field with appropriate boundary conditions at the moving boundary. The dynamics is then reduced to a free-boundary problem which is similar to the classic problem of viscous fingering in Hele-Shaw cells[4]. Similarly, the laplacian nature of the problem allows to take advantage of conformal mapping techniques.

One of the central results of this work is the proof of an exact expression for the instantaneous velocity of the center of mass (R_A) that establishes and interesting connection between shape and motility,

$$\dot{R}_A = \frac{v_p L}{A}\left(R_L - R_A\right), \tag{1}$$

where R_A and R_L are the center of masses of the area and of the contour, respectively. A stands for the area and L for the perimeter. This identity establishes a direct connection between the instantaneous velocity of the center of mass and simple geometrical properties of the contour. In other words, regardless of the actual evolution of the shape, at any time we can obtain the instantaneous velocity of the center of mass from the shape. This reduces the controversial question on the existence of steady motile shapes in this problem to the existence of asymmetric steady shapes, a problem that is more amenable to analytical and numerical treatment. With the help of conformal mapping techniques, we prove numerically that such solutions exist and some of them are stable.

Extending a previous linear stability analysis of the circular shape[2], we also show that the mechanism to initiate motion through symmetry breaking is necessarily nonlinear. To pursue the nonlinear character of the motility mechanism we also perform the center manifold reduction of the dynamics close to different bifurcation points, unraveling a rather complex mathematical structure. In particular, we find that there is an infinity of branches of traveling solutions that can be accessed analytically. Remarkably, the velocity of these solutions vanishes in the center manifold. With the help of high-precision arithmetics (64 digits), we have shown numerically that the velocity of these solutions as one approaches the bifurcation point is actually exponentially small, with the general form $\dot{R}_A \propto \exp(-a/g_i^n)$, where g_i is the distance to the bifurcation point of the $i-th$ mode. For $i = 2$, we have $a \approx 20.6$, and $n \approx 1/6$. This is a remarkable example of asymptotics beyond all orders that is reminiscent of the one associated to the steady-state selection mechanism in viscous fingering[4]. Whether this analogy entails a deeper connection is yet to be elucidated.

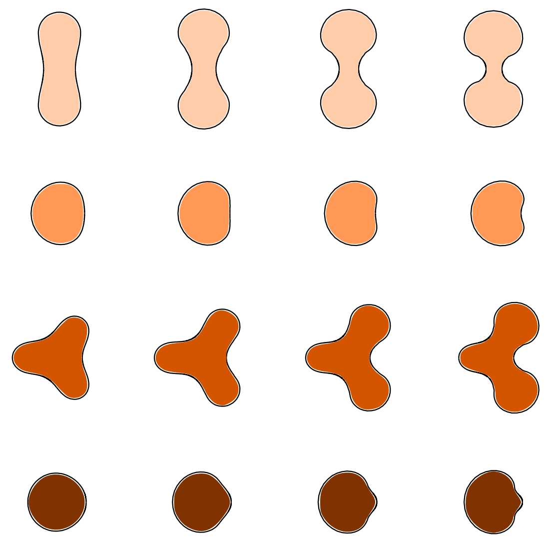

FIG. 1. Sample of motile steady shapes.

* jaume.casademunt@ub.es

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