

Nonlinear methods in modified gravities and the corresponding cosmologies

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Dark Friedman-Robertson-Walker fluids governed by non-linear inhomogeneous equations of state (EoS) have been considered that can be viewed as a conveniently simple paradigm for a whole class of models that exhibit phase transitions from a non-phantom towards a phantom era in the universe evolution. Such dark fluid models may also describe quintessence-like cosmic acceleration. Thermodynamical considerations for the processes involved are of great importance in the characterization of the global evolution of the corresponding universe. In particular, viscous cosmology models have been increasingly popular lately. From a hydrodynamical viewpoint this is quite a natural development, as the inclusion of the viscosity coefficients (shear and bulk) means physically that one departs from the case of an ideal fluid and incorporates the deviations from thermal equilibrium to first order. The case of an ideal (nonviscous) fluid is, after all, quite an idealized model. Also, under boundary-free conditions—such as in free turbulence—viscosities are physically most important. In a cosmological context, as the cosmic fluid is assumed to be spatially isotropic the shear viscosity is usually left out. One place where the appearance of bulk viscosity in the cosmic fluid should be expected to play an important role, is in the Big Rip phenomenon, i.e. the singularity of the universe in the future. This means that one or more of the physical quantities go to infinity at a finite time t in the future. Mathematically this implies divergent integrals, typically met when one uses the Friedmann equations to express t as an integral over the density ρ . A novel scenario has been proposed, the so-called Little Rip, models in which the nonviscous dark energy density increases with time (EoS parameter $w < -1$), but $w \rightarrow -1$ asymptotically, in that way avoiding the future singularity. Typically, this was found to occur when the scale factor increases rapidly with time, like $a(t) \sim \exp[\exp(t)]$ or higher exponentials. And this brings us to the topic, namely to examine the consequences of endowing the fluid with a bulk viscosity.

Actually, dark energy of phantom or quintessence nature with an equation of state parameter w almost equal to -1 often leads the universe evolution to a finite-time future singularity. An elegant solution to this problem has been recently proposed under the form of this Little Rip cosmology which appears to be a realistic alternative to the Λ CDM model¹. Whereas generically bulk viscosity tends to promote the Big Rip, we find that there are a number of situations where this is not the case and where the formalism nicely adjusts itself to the Little Rip scenario. In particular, a viscous fluid (or, equivalently, one with an inhomogeneous (imperfect) equation of state) is actually able to produce a Little Rip cosmology

as a purely viscosity effect. The possibility of its induction as a combined result of viscosity and a general (power-like) equation of state is investigated in detail. A physical interpretation of the dissolution of bound structures in the Little Rip cosmology is presented too.

In a related cosmological context, let us mention the known fact that, once the initial inflationary stage in the universe evolution, which originated a supercooling effect, ended up, temperature started to increase until, eventually, a recombination of elementary particles to create bound structures, in particular the first atoms, took place. This increase in the temperature is called re-heating, which also gives name to this period in the evolution of the universe, which subsequently led to the formation of the first stars, galaxies and much bigger structures, typical of the present universe at large scale.

One of the problems which appears in this process, when one tries to go the details of a feasible fundamental theory, as superstring theory, is the so-called field moduli problem. Field moduli can be considered, at first instance, as additional degrees of freedom corresponding to, say ‘pulsations’ of the universe when it was very small. They are a remnant of the compactification processes of superstrings, in going from the 11 dimensions of the mother theory to the 4 dimensional space-time we observe now.

In a seminal paper published a decade ago, Felder, Kovman and Linde² used a model of light particle production during and after the inflation process and were able to prove that, when implementing the model in string theory, a large amount of moduli fields were produced, what was in contradiction with astronomical observations. This is the so-called ‘moduli problem.’ In an attempt at trying to solve this issue, they obtained a reheating temperature of the universe that was abnormally small, far from the one predicted in theoretical models. On the contrary, in a more recent paper³, trying to solve the same problem by using a different argumentation, an abnormally too high reheating temperature has been postulated. Recently, we have given a solution to the moduli problem by a different method, which in the end nicely reconciles the above discrepancies and yields a very reasonable value for the reheating temperature⁴.

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⁴ J. de Haro and E. Elizalde, *PRL* **108**, 061303 (2012).