Strong anisotropy in surface kinetic roughening: theory and experiments

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It is well known that surfaces subject to growth or erosion processes, as in thin-film production, solid fracture, etc. display a self-affine behavior that can be studied through appropriate observables in real space (roughness and correlation functions) or in Fourier space (power spectral density, PSD). Such observables typically exhibit power-law behavior whose exponents characterize the long time and large length-scale behavior of the system under consideration.[1]

When the physical phenomena driving the growth or erosion process do not depend on the substrate direction, we are in the presence of isotropic scaling, the behavior of which is in general well described by the well known Family-Vicsek Ansatz.[1] However, there are many cases in which the growth of the surface depends on the direction considered along the substrate. Examples include the erosion of a tilted landscape by the action of wind and rain,[2] interfaces arising in simplified models of running sand-piles in the context of self-organized criticality,[3] or thin film growth[4,5] or erosion.[6,7] In these cases, the flux of material follows a preferred direction so that dynamics is not the same in the directions parallel and perpendicular to the external flux. Considering a two-dimensional surface, one has to define different sets of exponents associated with the different substrate directions.

In the specific context of thin film production in the presence of anisotropy, it is important to understand the relationship between the scalings of the two-dimensional PSD of the surface and the PSD of one-dimensional cuts along the directions parallel and perpendicular to the flux. This is due to the fact that many experimental setups are often designed to measure either the former (using e.g. X-ray diffraction), or the latter (using AFM or STM microscopies),[5] and therefore a quantitative comparison with theoretical models cannot be done unless this relationship is fully understood. On the other hand, from a theoretical point of view little is known about the conditions required for an interface equation that describes the evolution of a rough surface, to display anisotropic scaling in the asymptotic state (*strong anisotropy, SA*).

In this work, we formulate an anisotropic scaling Ansatz that clarifies the relationship among one and two-dimensional PSDs, and with correlation functions in real space. Our Ansatz is exact for *linear* equations with SA, and is useful to understand the long-time and large length-scale behavior of nonlinear equations, such as the one put forward by Hwa-Kardar (HK) in the context of running sand piles.[3] We moreover validate this hypothesis against experimental data from surface nanopatterning of silicon targets by ion-beam sputtering, both in the morphologically stable[7] and unstable situations.[6]

Additionally, using Dynamic Renormalization Group analysis and direct numerical simulations, we study from a theoretical point of view the appearance of *strong anisotropy* in nonlinear stochastic equations, both with conserved[3,8] and non conserved[4] dynamics. Our preliminary conclusions suggest that, in order to take place, asymptotic SA requires special (non-generic) conditions from the shape of the dynamical equation.

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