Spatiotemporal Bounded Noises in the Ginzburg-Landau model

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first important result of these investigations has been the emergence of noise-induced phase transitions "disorder to order" in spatially extended systems: in those cases a key bifurcation parameter is the force of strength of the spatial coupling (e.g. in the case of Laplace coupling, the diffusion coefficient). The most of the works in the field of noise-induced phase transitions are based on Gaussian noise (GN) or, more in general, white noises, in which is absent any spatiotemporal correlation or structure. This kind of fluctuations is appropriate when modelling system internal "hidden" degrees of freedom, of microscopic dynamical nature. On the contrary, fluctuations originating externally to the system in study may exhibit both temporal and spatial structure. Zero dimensional systems with colored Ornstein-Uhlenbeck noise (OU) showed to possess correlation-dependent properties that are missing in case of null autocorrelation, such as the emergence of stochastic resonance and re-entrant phase transitions[1]. Spatially extended systems exhibit even more striking effects when they are perturbed by spatially white but temporally colored noises: a complex interplay between noise intensity, spatial coupling and autocorrelation time[1].

Garcia-Ojalvo *et al* introduced in the spatial version of the OU noise, characterized by both a temporal scale τ and by a spatial scale λ[2]. Later, in a study on additive spatiotemporally colored perturbations in the Ginzburg-Landau field, they showed the existence of a nonequilibrium phase transition controlled by both the correlation time and the correlation length[3].

Recently a vast body of research focused on an important class of non-Gaussian stochastic processes: the bounded noises. The interest on bounded noises is motivated by the fact that in many applications noise models are inadequate because of their infinite domain: this should preclude their use to model stochastic fluctuations affecting parameters which must be bounded by physical constraints, especially in biology, where some parameters and quantities must be strictly positive. As a title of example, a GN-based modelling of the fluctuations affecting the pharmacokinetics of an antitumour drug delivering could lead to the paradox that the probability that the drug increases the number of tumour cells may become nonzero[4].

In our contribution we define two simple families of spatiotemporally bounded noises, which extend two kinds of temporal bounded noises frequently employed in literature: the Tsallis-Borland noise and the Cai-Lin noise[5,6,2]. These can be characterized in terms of the spatial coupling parameter λ and of the temporal correlation parameter τ on the distribution of the noise with suitable statistical observables and distributions. Differently from unbounded noise, bounded noises preserve their equilibrium distribution under tuning in τ, while in some noise cases λ modulation changes their shape. These characterization could be determinant when bounded noise are applied to dynamical systems, in particular in the presence of noise induced phase transitions.

Then we employed these two kinds of noises to study the phase transitions of the Ginzburg-Landau model, which under bounded perturbations resulted to exhibit a phenomenology quite different from the one induced by colored unbounded noises. Here, an increasing of noise temporal correlations enhances the "quenchedness" of noise, eventually producing an order to disorder transition, in an inverse manner with respect to what previously observed in former works with unbounded noise. On the other hand spatial coupling induces competitive effect on noise spatiotemporal fluctuations, resulting for some kind of noises a GL reentrant transition (order/disorder/order).

We point out finally that there is a dependence of the transitions or of stochastic resonance on the specific model of noise that has been adopted, and then in absence of experimental data on the distribution of the stochastic fluctuations for the problem in study, it is often necessary to compare multiple kinds of possible stochastic perturbations models.

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[1] H.S. Wio and K. Lindenberg, Modern Challenges in Statistical Mechanics, AIP Conference Proceedings, Vol. 658, pp. 1-60 (2003).

[2] J. Garcia-Ojalvo, J.M. Sancho and I. Ramirez-Piscina, Phys. Rev. E 46, 4670 (1992)

[3] J. Garcia-Ojalvo, J.M. Sancho, Phys. Rev. E 49, 2769 (1994)

[4] A. dÓnofrio and A. Gandolfi, Phys Rev E 82 (6), 061901 (2010)

[5] L. Borland, Phys. Lett. A 245 (1-2), 67 (1998)

[6] C.Q. Cai and Y.K. Lin, Phys Rev E 54 (1), 299 (2004)