Effects of topology on the one-dimensional Kardar-Parisi-Zhang universality class

Javier Rodriguez-Laguna[†,§,♯*], Silvia N. Santalla[†,‡], Rodolfo Cuerno[†,♯]

[†] *Grupo Interdisciplinar de Sistemas Complejos (GISC), Universidad Carlos III de Madrid*

The mathematical problem of modeling the evolution of a non-equilibrium rough interface arises for a huge variety of physical processes, such as thin film growth, fluid dynamics, flame-front propagation or undifferentiated biological growth.[1] In many relevant situations, the dynamics is characterized by the so-called Kardar-Parisi-Zhang (KPZ) universality class.

For the case of one-dimensional (1D) interfaces, this KPZ class is receiving an increasing attention lately. The reason is that not only the critical exponents are universal, but also the probability distribution for the fluctuations.[2] Such functions turn out to depend strongly on the topology of the interface in the embedding 2D space. In the so-called band-geometry, the interface is a closed curve wrapping around a cylinder. In this case, the height fluctuations follow the Tracy-Widom distribution associated with the Gaussian orthogonal ensemble (GOE) from random matrix theory.[4] The circular growth geometry takes place when the embedding space is just the Euclidean plane, the interface being a simple closed curve. In this case, the probability distribution for the interface radius corresponds to the Gaussian unitary ensemble (GUE), see [3] and references therein.

Recent experimental results[3] provide support for these results, which arise from exact solutions of the KPZ equation or from numerical simulations of discrete models. Unfortunately, different theoretical models or *ad-hoc* initial conditions are used for the analysis of the band-geometry and the circular geometry cases.

We propose a single stochastic partial differential equation defined on a 2D embedding space using only intrinsic geometry, i.e.: all terms in the equation are covariant. Thus, no distinction between the growth and the tangent directions to the interface are made *a priori*. The only difference between the band-geometry and the circular growth modes corresponds to the boundary conditions, namely, to the topology of the relation between the interface and the underlying space.[5] The equation reads

$$v_n(\vec{r}) = A_0 + A_1 K(\vec{r}) + A_2 \nabla^2 K(\vec{r}) + A_n \eta(\vec{r}), \qquad (1)$$

where v_n is the velocity along the local normal direction at any point of the interface, $\vec{r}$, K is the local curvature, ∇^2 is the Laplace-Beltrami operator, and η is a zero average, Gaussian white noise, uncorrelated both in space and time. Thus, the constants A_0, A_1, A_2, and A_n are free parameters.

Solving this single equation in the band and circular settings, we obtain profiles like those in Fig. for the former, and those in Fig. for the latter. We will show how the statistics of these profiles provide insight about the effects of topological constraints on the 1D KPZ universality class, that can thus be studied within a single framework.

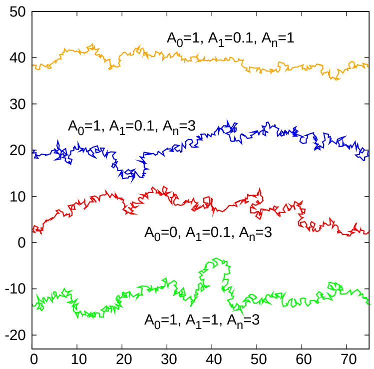

FIG. 1. Snapshots of the evolution of an interface described by Eq. (1) in the band-geometry setting.

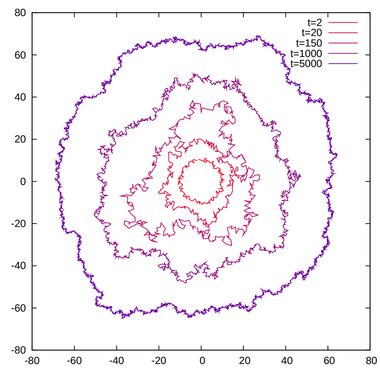

FIG. 2. Snapshots of the evolution of an interface described by Eq. (1) in the circular-geometry setting.

* javier.rodriguez@icfo.es

§ ICFO–Institute of Photonic Sciences, Castelldefels, Barcelona

♯ Departamento de Matemáticas, Universidad Carlos III de Madrid

‡ Departamento de Física, Universidad Carlos III de Madrid

[1] A.-L. Barabási and H. E. Stanley, *Fractal concepts in surface growth* (Cambridge University Press, Cambridge, 1995).

[2] T. Kriecherbauer and J. Krug, J. Phys. A: Math. Theor. **43**, 403001 (2010).

[3] K. A. Takeuchi, M. Sano, T. Sasamoto and H. Spohn, Sci. Rep. **1**, 34 (2011).

[4] P. Calabrese and P. Le Doussal, Phys. Rev. Lett. **106**, 250603 (2011).

[5] J. Rodriguez-Laguna, S.N. Santalla, R. Cuerno, J. Stat. Mech. P05032 (2011).