Quasiperiodic graphs: structural design, scaling and entropic properties

B. Luque, A. M. Núñez and J. P. Gómez
Dept. Matemática Aplicada y Estadística. ETSI Aeronáuticos, Universidad Politécnica de Madrid, Spain.

A novel class of graphs, here named quasiperiodic, are constructed via application of the Horizontal Visibility algorithm to the stationary trajectories of the universality class of low-dimensional nonlinear iterated maps with a cubic inflexion point, as represented by the circle map

$$\theta_{t+1} = f_{\Omega,K}(\theta_t) = \theta_t + \Omega - \frac{K}{2\pi}\sin(2\pi\theta_t),\ \mathrm{mod}\ 1 \quad (1)$$

along the quasiperiodic route to chaos. We show how the hierarchy of mode-locked regions represented by the Farey tree is inherited by their associated graphs. We are able to establish, via Renormalization Group (RG) theory, the architecture of the quasiperiodic graphs produced by irrational winding numbers with periodic continued fractions. And finally, we demonstrate that the RG fixed-point degree distributions are recovered via optimization of a suitably defined graph entropy.

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[2] Luque, B., Lacasa, L., Ballesteros, F.J., Robledo, A., *PLoS ONE* **6**, e22411 (2011).
[3] Luque, B., Lacasa, L., Ballesteros, F.J., Robledo, A., *Chaos*, **in press**, 2012.
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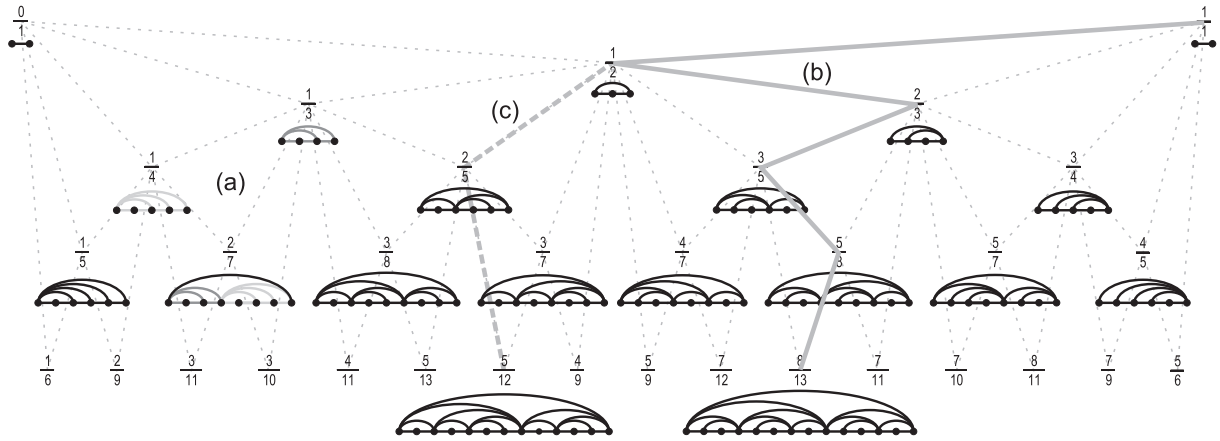

FIG. 1.: Six levels of the Farey tree and the periodic motifs of the graphs associated with the corresponding rational fractions p/q taken as dressed winding numbers ω in the circle map (for space reasons only two of these are shown at the sixth level).