# Microcanonical multifractal analysis of electric potential maps on the heart surface 

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Some characterizations of heartbeat dynamics as stochastic processes has been done as effective dynamics, with variability fluctuations following a multifractal distribution ${ }^{1,2}$. Such characterizations have shown applicability as automatic tools to help diagnosis of some arrhythmias, but very specific non-linear signal-analysis methods become necessary to tackle the whole dynamical complexity of the cardiac electrical activity.

In this context, methodological frameworks such as the Microcanonical Multifractal Formalism provide appropriate techniques for such characterization e.g., by means of the singularity exponents of the signals ${ }^{3-5}$. Singularity exponents correspond to the (fractional) leading order of expansion of a given multiscale measure on the signal. They are as such concepts of functional analysis independent of any particular model for the data, and they can be accurately calculated for every point of an empirical discrete signal.

The use of singularity exponents to characterize heartbeat dynamics has been successfully applied to skin electric potential measures from electrocardiograms and also catheter electrode measures on endocardial cavities ${ }^{6}$, where it has permitted to identify and characterize arrhythmic regimes like atrial fibrillation.Nevertheless it is possible to go further and characterize not only time series of electric potential but also time sequences of spatial maps of electric potential on the heart, with singularities evaluated both in space and time domain. Additionally, these types of data can be obtained in a non-invasive way (which drastically increases their clinical value) from epicardial projections of the electric potential obtained by solving the inverse problem of potential measures around the chest: a vest containing a grid of ECG electrodes measures the potential maps and from them they are projected on the epicardial surface.

This inversion is an ill-posed problem and several assumptions are needed to regularize the reconstructions. The robust estimation of dynamical transitions as singularity exponents can be used in such regularizations and help improve the information on areas of abnormal activity. We derive an effective cardiodynamical description with a simple and fast orientational dynamics modulated by a slow, more complex field. Gradient orientation on the most singular points is compatible with a stochastic process without memory. Modulation of this singularity orientation defines a complex but slow dynamics called source field ${ }^{7}$ and defined as the Radon-Nykodim derivative between a measure on the signal and another one on the orientation. The singularity degree at a point in
a signal is conceptually linked to how rare or unreconstructible is the value at that point from the rest of the signal, therefore the reconstruction is required to preserve these dynamical transitions. This can potentially be used to help automatic diagnosis and as a guide to catheter ablation procedures.


FIG. 1. Average source field on the epicardial surface of the heart atria and position of the mitral and tricuspid valves, for a case of a patient with atrial flutter.

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