Unveiling the complex organization of recurrent patterns in spiking dynamical systems

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Nature presents many fascinating complex systems in which distinguishing signatures of determinism in their high-dimensional dynamics is extremely challenging, not only because of the presence of noise, but also, because of lack of information about the system: one can measure only one or few relevant variables, and with a limited spatial and/or temporal resolution. A successful approach for studying such systems is by focusing on an event-level description of their dynamics, considering, for example, intervals between events. Examples of this approach include neuronal inter-spike-intervals, heart beat-to-beat intervals, earthquake waiting times, intervals between peak communication activity in social networks, activity of variable stars, etc. For the analysis of these events, a popular symbolic method, known as ordinal analysis[1,2], considers the relative order in which the events occur.

Here we use ordinal analysis to study the spiking output of a stochastic optical system, consisting of a semiconductor laser with feedback from an external reflector. Close to the lasing threshold, moderate feedback levels induce apparently random spikes in the laser output intensity, which become more frequent as the laser pump current is increased. This spiking output (see figure) has been referred to as low-frequency-fluctuations (or dropouts) and has attracted a lot of attention in the last decades[3], not only because of potential applications of optical chaos, but also, because the mechanisms triggering the spikes involve the interplay of nonlinearity, delay and noise, which are ubiquitous in nature.

We analyze experimentally recorded sequences of spikes by transforming the time trace into a sequence of ordinal patterns, or words. We unveil a nontrivial organization of ordinal patterns underlying the sequence of optical spikes. It is shown a hierarchical and clustered organization of the ordinal patterns, with a well defined structure of the probabilities of occurrence. Simulations of the Lang and Kobayashi (LK) model[4] are in good agreement with the observations. To the best of our knowledge, in spite of the large attention that the optical spikes have attracted over the years, ours is the first demonstration of an organized structure of spike patterns.

Most importantly, we identify a minimal iterative model, a modified circle map (proposed by Neiman et al.[5] to represent spike correlations in sensory neurons) that, despite its simplicity, displays a symbolic dynamics with the same hierarchical and clustered organization of patterns as the experimental data. In order to confirm that this minimal model indeed represents the symbolic dynamics underlying the sequence of optical spikes, we consider the spiking output of the laser with periodic forcing (via a direct modulation of the pump current). We demonstrate that the symbolic dynamics underlying the spikes emitted by the forced laser is also in good qualitative agreement with the symbolic dynamics of the modified circle map.

Since the circle map is a generic model, we expect that our finding will apply also to other dynamical systems. Moreover, our results suggest that optical neurons inspired by biological ones could be built using semiconductor lasers, and could provide a controllable set up to mimic neuronal activity. Our findings could also pave the way for novel neuro-inspired optical computing devices.

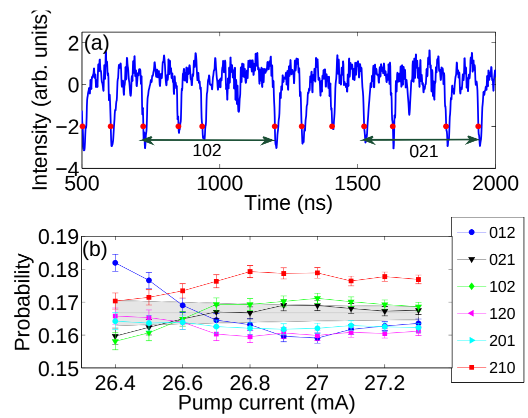

FIG. 1. (a) Experimental time trace of the laser intensity. The red dots indicate the spike times and two ordinal patterns (words) are indicated as examples. (b) Probabilities of the different patterns versus pump current.

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