Random walkers that die as they move

S.B. Yuste[†], E. Abad[*], and K. Lindenberg[‡]
Centro Universitario de Mérida
Departamento de Física Aplicada, Universidad de Extremadura
06800-Mérida (Badajoz)

In statistical physics, random walks are widely used to study stochastic transport in a large variety of experimental systems. Exploration and trapping properties of walkers that do not undergo reactions or transformations have been comprehensively studied by many authors. However, the corresponding properties of reactive walkers have received considerably less attention. In particular, work involving the arrival statistics of so-called mortal or evanescent walkers, that is, walkers that die in the course of their motion, is surprisingly scarce in spite of the obvious ubiquity of death and death-like processes in many real problems in physics, biology and chemistry.

As it turns out, we have found a few examples where *ad-hoc* models were developed and tailored to study particular experimental situations (e.g., photon scattering accompanied by absorption in a tissue). However, the fact that the effect of elementary death processes on the exploration properties of random walkers can be investigated using arguments and techniques similar to those used for conventional, immortal walkers has, until recently, gone largely unnoticed. As a result of this observation, we have been able to develop an overarching theoretical framework allowing one to obtain the most relevant properties of mortal walkers[1]. This achievement seems very timely to us, as the presence of death processes leads to a completely new physics in the transport behavior of diffusing particles, and it also has potential relevance for a wealth of experimental problems. These problems touch on fundamental questions as diverse as the relaxation behavior of defects in dielectric media or the extent of the region where a diffusing radioisotope remains potentially emissive.

We aim to present in a pedagogical way some results that have been published recently[1,2]. To begin with, we shall focus on exploration and trapping properties in discrete time, such as the average number S_n^* of distinct sites visited by a mortal walker on perfect lattices of arbitrary dimension after a certain number of time steps n (at each time step the walker is assumed to perform a jump). Let us denote by S_n the counterpart of S_n^* for an immortal walker, that is, a walker that lives forever. The quantity S_n is known to diverge with n in all dimensions. However, if the walker has a constant probability of dying $1-e^{-\lambda}$ in each time step, S_n^* tends to a *finite* value expressible in terms of λ and the generating function of S_n (see, e.g., Fig. 1, displaying the behavior of S_n^* in the one-dimensional case).

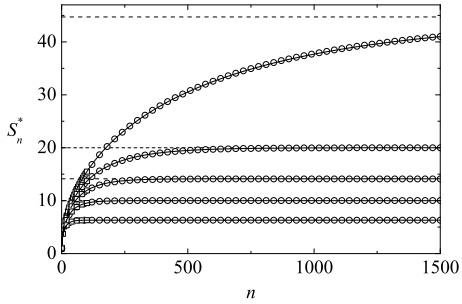

FIG. 1. Average number S_n^* of distinct sites visited by a Markovian mortal walker vs. the time step number n. The walker performs nearest neighbor jumps on an infinite $1d$ lattice as long as it survives. The curves correspond, from top to bottom, to $\lambda = 0.001, 0.005, 0.01, 0.02$ and 0.05 ($e^{-\lambda n}$ is the probability that the walker has survived after n steps). Solid lines give analytic results and broken lines give analytic asymptotic values; circles, simulation values; squares, exact values obtained from the first 100 coefficients in the ξ power expansion of the generating function $S^*(\xi)$ associated with S_n^*.

We shall also consider the continuous time limit of the above system, namely, the behavior of the Wiener sausage defined by a mortal random walker as a function of time[2], a key quantity for the computation of reaction rates in chemical kinetics. We shall also address memory effects, that is, the behavior of the average number of distinct sites visited by an evanescent continuous time random walker up to a given time[3]. The above results are expected to be relevant for problems involving diffusing particles subject to photon emission or radioactive decay, or stochastically moving prey hunted by a collection of predators, to name but a few.

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[*] E-mail: eabad@unex.es
[†] Facultad de Ciencias, Departamento de Física, Universidad de Extremadura, Avda. de Elvas s/n, E-06071, Badajoz.
[‡] Department of Chemistry and Biochemistry, and BioCircuits Institute, University of California San Diego, 9500 Gilman Drive, La Jolla, CA 92093-0340, USA.

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