New spreading law of thin film liquids controlled by gravity and vdW forces under thermal fluctuations

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The dynamics of a stochastic thin film is given by the equation

$$\frac{\partial h}{\partial t} = \nabla \cdot \left(h^3 \nabla p\right) + \sqrt{2\sigma}\,\nabla \left[h^{3/2}\epsilon(x,t)\right] \qquad (1)$$

where p is a generalized pressure term accounting for the effects of surface tension, gravity and van der Waals (vdW) attraction. The second term on the RHS represents thermal fluctuations where $\epsilon(x,t)$ is a Gaussian white noise, zero mean and delta correlated.[1,2] When surface tension is dominant force (no vdW, gravity nor noise) any perturbation of a film as well as the droplet spreading will lead to a flat film solution. In the case of droplet spreading, the droplet width ℓ, see the fig. 1, undergoes Tanner's law[3], $\ell \sim t^{1/7}$. If thermal fluctuations are accounted, it was shown[4] that the stochastic force will take over the dynamics of the system and introduce a new enhanced spreading low $\ell \sim t^{1/4}$.

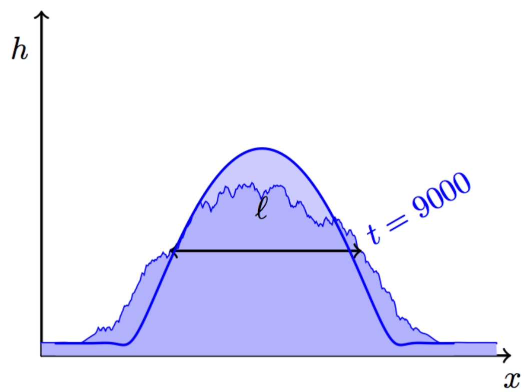

FIG. 1. Stochastic and Deterministic droplet surface. Droplet width ℓ describes dynamics of a droplet. Results from simulations.

On the other hand, when we have both surface tension and vdW force the stable solution will be a droplet with a contact angle formed[5] as a result of the interplay between these two forces. The contact angle can easily be calculated using Laplace-Young condition.[6]

Here we present results from simulations of the $1d$ stochastic thin film equation 1 using finite differences scheme,[7] in which noise competes with the effects of vdW and surface tension forces which introduce a fixed contact angle. On the fig. 2 we show that on average there is a small change in droplet width in the region where the width has saturated when the noise term is introduced. The noise term together with surface tension tends to spread the droplet which leads to a small correction in the Laplace-Young condition.

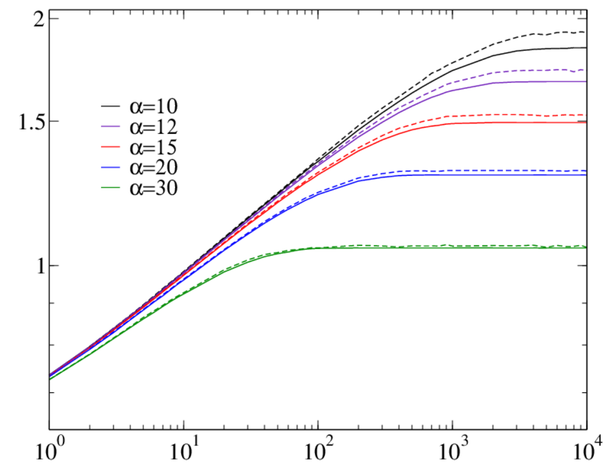

FIG. 2. Width of a droplet as a function of time for a given contact angle. Dashed lines represent stochastic films while continuous lines represent deterministic films.

Moreover, fluctuations accelerate breakup when a thin flat film is slightly perturbed[1,6] the final aim of the work being the study of the dynamics of such films in more realistic $3D$ setups.

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