Mapping dynamics to graphs. The visibility algorithm.

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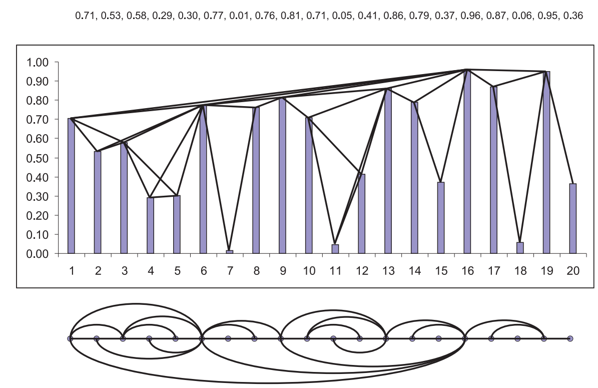
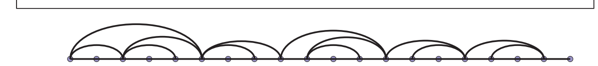
Disregarding any underlying process (and therefore any physical, chemical, economical or whichever meaning of its mere numeric values), we can consider a time series just as an ordered set of values and play the naive mathematical game of turning this set into a different mathematical object with the aids of an abstract mapping, and see what happens: which properties of the original set are conserved, which are transformed and how, what can we say about one of the mathematical representations just by looking at the other... This exercise is of mathematical interest by itself. In addition, it turns out that time series or signals is a universal method of extracting information from dynamical systems in any field of science. Therefore, the preceding mathematical game gains some unexpected practical interest as it opens the possibility of analyzing a time series (i.e. the outcome of a dynamical process) from an alternative angle. Of course, the information stored in the original time series should be somehow conserved in the mapping. The motivation is completed when the new representation belongs to a relatively mature mathematical field, where information encoded in such a representation can be effectively disentangled and processed. This is, in a nutshell, a first motivation to map time series into networks.

This motivation is increased by two interconnected factors: first, although a mature field, time series analysis has some limitations, when it refers to study the so called complex signals. Beyond the linear regime, there exist a wide range of phenomena (not exclusive to physics) which are usually embraced in the field of the so called Complex Systems. Under this vague definition lies a common feature: the relevant effect of nonlinearities in their mathematical representation. This feature can be reflected in the temporal evolution of (at least one of) the variables describing the system and necessitates the use of specific tools for nonlinear analysis. Dynamical phenomena such as chaos, long-range correlated stochastic processes, intermittency, multifractality, etc... are examples of complex phenomena where time series analysis is pushed to its own limits. Nonlinear time series analysis develops from techniques such as nonlinear correlation functions, embedding algorithms, multifractal spectra, projection theorems... tools that increase in complexity parallel to the complexity of the process/series under study. New approaches, new paradigms to deal with complexity are not only welcome, but needed. Approaches that deal with the intrinsic nonlinearity by being intrinsically nonlinear, that deal with the possible multiscale character of the underlying process by being designed to naturally incorporate multiple scales. And such is the framework of networks, of graph theory. Second, the technological era brings us the possibility of digitally analyze myriads of data in a glimpse. Massive data sets can nowadays be parsed, and with the aid of well suited algorithms, we can have access and filter data from many processes, let it be of physical, technological or even social garment. It is now time to develop new approaches to filter such plethora of information.





It is in this context that the network approach for time series analysis was born. The family of visibility algorithms (Va) constitute one of other possibilities to map a time series into a graph and subsequently analyze the structure of the series through the set of tools developed in the graph/complex networks theory.

The idea of mapping time series into graphs seems attractive because it lays a bridge between two prolific fields of modern science as Nonlinear Signal Analysis and Complex Networks Theory, so much so that it has attracted the attention of several research groups which have contributed to the topic with different strategies of mapping. Among all these methods of mapping, in this talk we will concentrate our attention on the one developed in[1] and subsequent works. To cite some of its most relevant features, we will stress its intrinsic nonlocality, its low computational cost, its straightforward implementation and its quite 'simple' way of inherit the time series properties in the structure of the associated graphs. These features are going to make it easier to find connections between the underlying processes and the networks obtained from them by a direct analysis of the latter.

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