A novel focus on population dynamics under mutualistic interactions

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Mutualistic interactions, which are beneficial for all the species involved, have an internal structure that makes them resilient to external perturbations. Actually, the relations between plants and their pollinators and seed dispersers are paradigmatic examples of mutualism. In this context, Ehrlich and Raven[1] alluded to the importance of plant-animal interactions in the generation of Earth's biodiversity. The simplest mutualistic model was proposed by May[2]. Each of May's equations for two species is a logistic model with an extra term accounting for the mutualistic benefit. It is the same idea as in the Lotka-Volterra model but interactions between species always add to the resulting population. May's equations for two species can be written as

$$\frac{dN_1}{dt} = r_1\, N_1\, \left(1 - \frac{N_1}{K_1}\right) + r_1\, N_1\, \beta_{12}\, \frac{N_2}{K_1}, \tag{1}$$

$$\frac{dN_2}{dt} = r_2\, N_2\, \left(1 - \frac{N_2}{K_2}\right) + r_2\, N_2\, \beta_{21}\, \frac{N_1}{K_2} \tag{2}$$

where $N_1\,(N_2)$ is the population of the species $1\,(2)$; $r_1\,(r_2)$ is the intrinsic growth rate of population $1\,(2)$ and $K_1\,(K_2)$ the carrying capacity. This is the maximum population that the environment can sustain indefinitely, given food, habitat, water and other supplies available in the environment. Finally, β_{12} is the coefficient that embodies the benefit for N_1 of each interaction with N_2. May's model major drawback is that it may lead to unbounded growth, but anyway it has been an inspiration for subsequent mutualistic models that incorporate terms to solve this problem.

In this work, we introduce a model for population dynamics under mutualism that preserves the original logistic formulation. It is mathematically simpler than the widely used type II models, although it shows similar complexity in terms of fixed points and stability of the dynamics. We perform an analytical stability analysis and numerical simulations to study the model behavior in more general interaction scenarios including tests of the resilience of its dynamics under external perturbations. Despite its simplicity, our results indicate that the model dynamics shows an important richness that can be used to gain further insights in the dynamics of mutualistic communities.

Our hypothesis is that mutualism is just another element that contributes to the species growth rate, that means r is a function of mutualism and not a third term added to the logistic equation. We choose a linear approach:

$$r_i^{new} = r_i^a + \sum_{k=1}^{n_p} b_{ki}^{pa}\, N_k^p \tag{3}$$

where the superscript p stands for *plants* and a for *animals*. Following this reasoning, the friction α must also take mutualism into account to avoid an unreallistic infinite benefit. To simplify the model, we assume that the effect is proportional to the benefit.

$$\alpha_i^{new} = \alpha_i^a + c_i^a \sum_{k=1}^{n_p} b_{ki}^{pa}\, N_k^p \tag{4}$$

Under these assumptions the system is described by this set of differential equations for n_p species of plants and n_a of animals:

$$\frac{1}{N_j^p}\frac{dN_j^p}{dt} = r_j^p + \sum_{l=1}^{n_a} b_{lj}^{ap}\, N_l^a - \left(\alpha_j^p + c_j^p \sum_{l=1}^{n_a} b_{lj}^{ap}\, N_l^a\right) N_j^p \tag{5}$$

The right term can be interpreted as an *effective growth rate.*

$$r_{eff_j}^p = r_j^p + \sum_{l=1}^{n_a} b_{lj}^{ap}\, N_l^a - \left(\alpha_j^p + c_j^p \sum_{l=1}^{n_a} b_{lj}^{ap}\, N_l^a\right) N_j^p \tag{6}$$

$$\frac{dN_j^p}{dt} = r_{eff_j}^p N_j^p \tag{7}$$

The *carrying capacities* are fixed points of system. It is easy to deduce that without mutualism K is the constant r/α. Under the presence of hypothetical infinite mutualism would be $1/c_i^a$. So, the coefficient we included in (4) is the inverse of the maximum population of the species i when the number of mutualistic individuals is so high that $c_i \sum_{k=1}^{m} b_{ki} N_k >> \alpha_i$.

Finally, we perform an analytical stability analysis and numerical simulations to study the model behavior in more general interaction scenarios including tests of the resilience of its dynamics under external perturbations. Despite its simplicity, our results indicate that the model dynamics shows an important richness that can be used to gain further insights in the dynamics of mutualistic communities[3]

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