Mapping dynamics to graphs. The case of intermittency.

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The type-I and type-II intermittency routes to (or out of) chaos are investigated within the Horizontal Visibility (HV) graph theory. For that purpose, we address both trajectories generated by unimodal maps close to an inverse tangent bifurcation and trajectories generated by iterated maps close to a Neimark-Sacker bifurcation and we construct their associated HV graphs. We show how the alternation of laminar episodes and chaotic bursts imprints a fingerprint in the resulting graph structure. Accordingly, we derive a phenomenological theory with analytical results that predict quantitative values for several network parameters. In particular, we predict that the characteristic power-law scaling of the mean length of laminar trend sizes is fully inherited by the variance of the graphs degree distribution, in good agreement with the numerics. We also report numerical evidence on how the characteristic power-law scaling of the Lyapunov exponent as a function of the distance to the critical point is inherited in the graphs by an analogous scaling of block entropy functionals defined on the graphs. Furthermore, we are able to recast the full set of HV graphs generated by both types (I, II) of intermittent dynamics into renormalization group frameworks, where the fixed points of thier graph-theoretical Renormalization Group (RG) flows account for the different types of dynamics. We also establish that the nontrivial fixed points of these flows coincide with the critical condition in each case and that the corresponding invariant graphs exhibit extremal entropic properties.

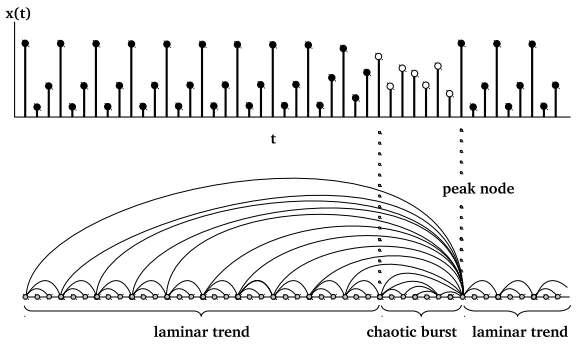

FIG. 1. Graphical illustration of how the Horizontal Visibility (HV) graph inherits in its structure the dynamics of the associated intermittent series. In the top of the figure we show a sample intermittent series generated by the logistic map close to μ_c ($\epsilon > 0$), producing laminar regions (black) mixed with chaotic bursts (white). In the bottom we plot the associated HV graph. Laminar regions are mapped into nodes with a periodic backbone, whereas the actual pseudoperiodicity of the series is inherited in the graph by the existence of so called peak or interfacial nodes. Chaotic bursts are mapped into chaotic nodes, with a characteristic degree distribution (see the text).

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[2] Núñez, Á. M., Lacasa L., Gómez J. P., *Journal of Physics A* **in press** (2014).