Fragility and robustness of the Kardar-Parisi-Zhang universality class

Matteo Nicoli*, Rodolfo Cuerno†, and Mario Castro‡
Department of Physics and Center for Interdisciplinary Research on Complex Systems
Northeastern University, 02115 – Boston, USA

One of the most powerful concepts in contemporary Statistical Mechanics is the idea of universality, by which microscopically dissimilar systems show the same large scale behavior, provided they are controlled by interactions that share dimensionality, symmetries, and conservation laws. In complex systems it becomes enormously simplifying, as significant descriptions can be put forward on the basis of the general principles just mentioned.

Celebrated non-equilibrium systems include those with generic scale invariance, displaying criticality throughout parameter space. Examples are self-organized-critical and driven-diffusive systems, or surface kinetic roughening. Indeed, the paradigmatic Kardar-Parisi-Zhang (KPZ) equation for a rough interface,

$$\partial_t h = \nu \nabla^2 h + \frac{\lambda}{2} (\nabla h)^2 + \eta(\mathbf{x}, t), \tag{1}$$

is very recently proving itself as a remarkable instance of universality. The exact asymptotic height distribution function has been very recently obtained for $d = 1$:[1] it is given by the largest-eigenvalue distribution of large random matrices in the Gaussian unitary (GUE) (orthogonal, GOE) ensemble, the Tracy-Widom (TW) distribution, for globally curved (flat) interfaces, as proposed in Ref. 2 and reviewed in Ref. 3. Beyond their fascinating connections with probabilistic and exactly solvable systems, these results are showing that, not only are the critical exponent values common to members of this universality class, but also the distribution functions and limiting processes are shared by discrete models and continuum equations,[4] and by experimental systems, from turbulent liquid crystals[5] to drying colloidal suspensions.[6]

In this work we assess the dependence on substrate dimensionality of the asymptotic scaling behavior of a whole family of nonlocal equations that feature the basic symmetries of the KPZ equation[7]

$$\partial_t h_{\mathbf{k}}(t) = (\nu k^{\mu} - \mathcal{K} k^2) h_{\mathbf{k}}(t) + \frac{\lambda}{2} \mathcal{F}[(\nabla h)^2]_{\mathbf{k}} + \eta_{\mathbf{k}}(t). \tag{2}$$

Even for cases in which, as expected from universality arguments, these models display KPZ critical exponent values, their behavior deviates from KPZ scaling for increasing system dimensions.[8] Such a fragility of KPZ universality contradicts naive expectations, and questions straightforward application of universality principles for the continuum description of experimental systems. Still, we find that the ensuing limit distributions coincide with those of the KPZ class in one and two dimensions (see Fig. 1), demonstrating the robustness of the latter under changes of the critical exponent values.

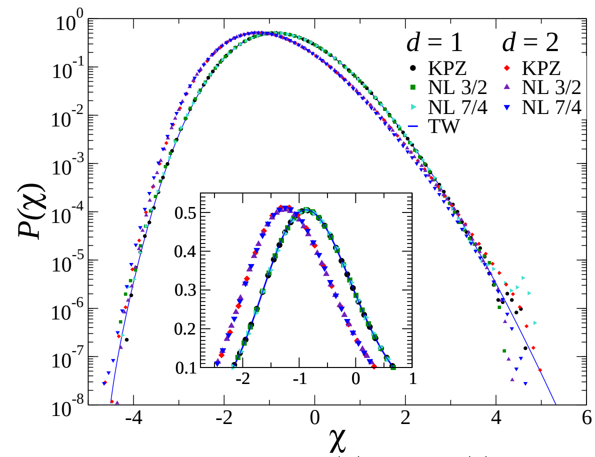

FIG. 1. Height distribution $P(\chi)$ for Eq. (1) and Eq. (2) for $\mu = 3/2,\ 7/4$ in case of one- and two-dim. substrates. The solid line is the TW-GOE distribution expected for $d = 1$.[9]

---

* m.nicoli@neu.edu

† Departamento de Matemáticas and Grupo Interdisciplinar de Sistemas Complejos (GISC), Universidad Carlos III de Madrid, 28911 – Leganés, Spain.

‡ GISC and Grupo de Dinámica No Lineal (DNL), Escuela Técnica Superior de Ingeniería (ICAI), Universidad Pontificia Comillas, 28015 – Madrid, Spain.

[1] T. Sasamoto and H. Spohn, Phys. Rev. Lett. **104**, 230602 (2010); G. Amir, I. Corwin, and J. Quastel, Commun. Pure Appl. Math. **64**, 466 (2011); P. Calabrese and P. Le Doussal, Phys. Rev. Lett. **106**, 250603 (2011).

[2] M. Prähofer and H. Spohn, Phys. Rev. Lett. **84**, 4882 (2000); Physica A **279**, 342 (2000).

[3] T. Kriecherbauer and J. Krug, J. Phys. A: Math. Theor. **43**, 403001 (2010); I. Corwin, Random Matrices: Theor. Appl. **1**, 1130001 (2012).

[4] S. G. Alves, T. J. Oliveira, and S. C. Ferreira, EPL **96**, 48003 (2011); T. J. Oliveira, S. C. Ferreira, and S. G. Alves, Phys. Rev. E **85**, 010601(R) (2012).

[5] K. A. Takeuchi and M. Sano, Phys. Rev. Lett. **104**, 230601 (2010); K. A. Takeuchi *et al.*, Sci. Rep. **1**, 34 (2011).

[6] P. Yunker *et al.*, Phys. Rev. Lett. **110**, 035501 (2013); *ibid.* **111**, 209602 (2013); M. Nicoli, R. Cuerno, and M. Castro, *ibid.* **111**, 209601 (2013).

[7] M. Nicoli, R. Cuerno, and M. Castro, Phys. Rev. Lett. **102**, 256102 (2009); J. Stat. Mech.: Theor. Exp. (2011) P10030.

[8] M. Nicoli, R. Cuerno, and M. Castro, J. Stat. Mech.: Theor. Exp. (2013) P11001.

[9] A. Edelman and P.-O. Persson, *Numerical Methods for Eigenvalue Distributions of Random Matrices*, arXiv:math-ph/0501068 (2005).