Interplay between internal time scales and network topology in coupled nonlinear oscillators

Jordi Zamora-Munt*, Manuel A. Matías, and Pere Colet
IFISC, Instituto de Física Interdisciplinar y Sistemas Complejos
CSIC-UIB, Universidad de las Islas Baleares 07122-Palma (Mallorca)

Interaction through a mediator is a robust mechanism to achieve a high degree of synchronization in large populations of oscillators. Some examples of this behavior include yeast cells in a common medium[1], pedestrians walking in a bridge[2] or star-coupled semiconductor lasers[3]. In this context a progressive transition from the incoherent state for active oscillators or a sudden transitions from the quiescent state for passive oscillators have been reported when the number of oscillators is above a critical value[4]. All those systems consider a very symmetric coupling, as all the oscillators see the same mean field as averaged by the hub. On the other hand, one expects that in real systems this mediated coupling through a common element coexists with direct interactions among the oscillators. However, in this context the interplay between these two types of interaction, mediated and direct, is an open problem.

Being our goal to consider the transitions induced by coupling in quiescent units to global oscillations, our model consists of N (synchronous) identical Landau-Stuart (LS) oscillators, z_j , coupled through a common linear damped oscillator or hub, F , and directly coupled by direct coupling:

$$\dot{z_j} = (\mu + i\omega)z_j - |z_j|^2 z_j + k_A^{(1)} (F - z_j) + k_B \sum_{k=1}^{N} B_{jk} z_k, \tag{1}$$

$$\dot{F} = (-\gamma + i\Omega)F + k_A^{(2)} \sum_{j=1}^{N} z_j. \tag{2}$$

where μ is the linear gain parameter, such that a supercritical Hopf bifurcation happens for $\mu = 0$ and ω is the oscillation frequency. We will consider the case $\mu < 0$, in which the oscillators are quiescent, case in which it has been shown that coupling through a common element, $k_B = 0$, induces a transition to a oscillatory synchronized state[2–4]. The hub acts as a bandpass filter where $\gamma > 0$ can be related to the filter bandwidth and Ω to the central frequency with maximum transfer function. The direct interactions can be attractive when B_{jk} is positive or respulsive when B_{jk} is negative. The entries B_{jk} defines the topology of the direct couplings, and k_A^i and k_B are positive coupling strengths.

Our goal is to analyze the combined effect of direct, i. e. $k_B \neq 0$, and hub mediated interactions among a set of identical LS oscillators. We have carried out this study using the Master Stability Function formalism for identical oscillators[5] recently generalized to consider synchronization between groups of oscillators[6]. The conditions for stable synchronization in the proposed system are obtained analytically with this formalism for any interaction topology.

Two different cases have been considered:

For a resonant hub ($\omega = \Omega$) we demonstrate that synchronization is stable for any direct coupling topology with $B_{jk} \geq 0$ for all j and k. In turn, if some B_{jk} are negative, i. e. for some repulsive interactions, a more complex scenario is observed. Synchronization is stable in a broad range of parameters. However, large repulsive coupling coefficients can destabilize the synchronized state. In this case, a rich variety of coexisting dynamics such as inhomogeneous amplitude synchronization or rotating waves of different orders become stable.

For a nonresonant hub ($\omega \neq \Omega$) and attractive direct coupling the scenario is similar to the resonant case. Stable synchronization is observed for any direct coupling topology while, a large enough frequency detuning leads to amplitude death of the oscillations. More interesting is the case when attractive and repulsive direct interactions coexist. We report the existence of stable synchronization in a narrow range of nonzero frequency detunings that can be interpreted as the resonance of the internal time scales of the dynamical units with the modes of the coupling topology.

Our study shows that the inclusion of a direct coupling topology in a system of oscillators initially coupled through a common passive medium has two substantially different cases depending on the sign of the interactions. When all the interactions are attractive, synchronization is reinforced by the direct coupling. However, when both attractive and repulsive couplings coexist, two opposite effects can be observed. Negative interactions can either destabilize the synchronous state or they can also have the opposite effect of stabilizing it. The latter being the result of a non-trivial interplay between the coupling topology and the dynamics of the oscillators.

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* jordi@ifisc.uib-csic.es

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