Pinning by noise

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Research on *localized patterns*—bubbles of *metastable* homogeneous states (HS), also called *pinning states* or "solitons"—has recently undergone an upsurge of interest. Especially puzzling are those arising in one-dimensional (1D) reaction–diffusion (RD) systems, since "inverse nucleation"-like arguments (invoking negative surface tension) break down. Moreover in *simple* 1D bistable RD systems, localized patterns *cannot* be stable because for equally stable HS, the solvability condition determining the time-derivative of the "soliton" width Δ can be interpreted as an "attracting force" between the kinks bounding the bubble, which make it shrink and disappear[1]. A route to stable localized patterns in 1D RD systems is to introduce a *nonlocal interaction* term[2].

Here we report another route, involving *two* essential ingredients[3]: i) an aggregating current $J_A(x,t)$ which can in certain situation (e.g. when due to an attracting potential with range $r_a \ll L_{diff}$) be assimilated to an "antidiffusive" term[4]; ii) a *multiplicative* noise of the kind leading to *entropic* noise-induced phase transition[5]. The dynamical equation of a field $0 < \phi(x,t) < 1$ is

$$\partial_t \phi = Q_\lambda(\phi) + \partial_x [D_{eff}(\phi)\partial_x \phi] \qquad (1)$$

with $Q_\lambda(\phi) := Q(\phi) - \lambda \frac{\mathrm{d}\Gamma}{\mathrm{d}\phi}$ accounting for ingredient (ii) and $D_{eff}(\phi)$ accounting for ingredient (i).

We aim to stabilize "solitons" when introducing as initial condition either

$$\phi_0^u(x) := 0.48[\tanh \rho A(x + x_f) - \tanh \rho A(x - x_f)] \text{ or}$$
$$\phi_0^d(x) := 1 - 0.48[\tanh \rho A(x + x_f) - \tanh \rho A(x - x_f)],$$

(with $\rho > 1$ and $A := L_{diff}/r_a$) according to whether we want to perturb the low-value (ϕ_d) or the high-value (ϕ_u) HS. (We take $x_f = 21,600\,\delta x$, with $\delta x = 2.510^{-4}$) Regarding the simplest multiplicative-noise factor $\Gamma^{1/2}(\phi)$ enabling the noise to destabilize only one HS, it comes out that

$$\Gamma_u := \exp[b(\phi - 1)] \text{ affects } \phi_u \text{ and}$$
$$\Gamma_d := \exp[-b\phi] \text{ affects } \phi_d$$

(we use $b = 10$). These factors put the affected HS around the critical point that designates his disappearance. This idea is not expected to work for too large noise intensities, since the solution that is intended to affect would cease to exist. Figure 1 (respectively Fig. 2) shows states where ϕ_d (respectively ϕ_u) has been pinned by the multiplicative noise. This numerical result is supported by a rough calculation that appeals to the solvability condition[1,2,7].

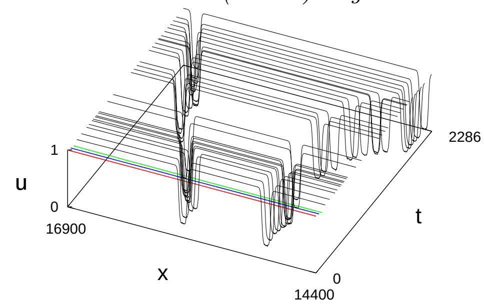

FIG. 1. System excited with $\phi_0^d(x)$. Evolution of two "solitons" with ϕ_d pinned.

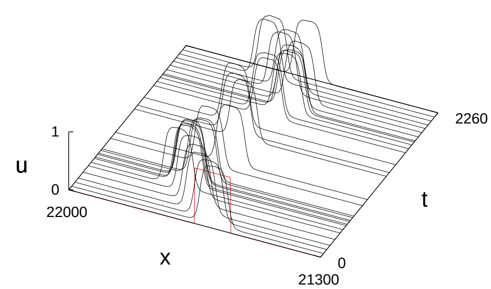

FIG. 2. System excited with $\phi_0^u(x)$. Evolution of one "soliton" with ϕ_u pinned.

We take for $Q(u)$ in Eq. (1) the normal form $Q(u) = (\beta - u^2)u + \mu$, with $u = \phi - 0.5$ and μ a small parameter, so that its stationary front solutions can be written as

$$u^s \approx u^s_\pm = \sqrt{\beta}\,\tanh\left[(\beta/2)^{1/2} A(x - x_f^\pm)\right].$$

The resulting equation for Δ shows that under certain conditions,
- with Γ_u, the only solution is stable S^+ solitons and
- with Γ_d, the only solution is stable S^- solitons.

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