Co-evolving link states dynamics

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Collective properties of interacting units have traditionally been studied considering that each of these units is characterized by a property or state, and interacts with others in a network of interactions. The result of the interaction depends on the state of the interacting units and the rules of the particular dynamics implemented. The typical example would be a spin system in a lattice, where there is a spin in each node with a given state and it interacts with its lattice neighbors in a way that depends on their relative spin state. However, there is a number of possible interactions, particularly when considering social systems, in which the state variable is more properly described as a state of the interaction link than a state of the interacting individuals. Typical examples are interactions such as friendship-enmity relationships[2], trust or the use of competing languages.

Motivated by this idea, J. Fernández-Gracia *et al.*[1] have studied a prototype model for the dynamics of the states of the links in a fixed network of interacting units. In their model, each link can be in one of two equivalent states and the dynamics implemented is a simple majority rule for the links, so that in each dynamical step the state of a randomly chosen link is updated to the state of the majority of its neighboring links. The authors find a broad distribution of possible asymptotic configurations, including both frozen and dynamically trapped configurations, some of which have no counterpart under traditional node dynamics in the same topologies and whose probability of appearance is significantly increased.

The study of co-evolving dynamics and topologies has also received much attention recently, particularly in relation to a social context. The most common finding in these coupled systems is a fragmentation transition, which usually occurs for a certain relationship between the time scales of both processes: node states are updated according to their neighbors' states, links between nodes are rewired according to their states. A large number of dynamics and rewiring rules have been studied[3],[4],[5], using different methods to characterize the resulting critical point.

Taking the model studied by J. Fernández-Gracia *et al.*[1] as our starting point, we develop a co-evolving dynamics by defining a certain rewiring mechanism inspired by the case of competing languages. We find that, for any value of the probability of rewiring, frozen disordered configurations are not stable, so the system fully orders. However, this ordering may lead to a one component consensus or to a fragmentation into two components, each one with different link states. We show that the critical point of this fragmentation transition tends to zero in the thermodynamic limit. We also present evidence of the different time scales in which the ordering process takes place depending on which mechanism is responsible for the ordering: majority rule or rewiring. Finally, we present an analytical treatment explaining the behavior of the system when the rewiring process is the leading mechanism.

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