Stochastic-resonant spatiotemporal patterns in a FitzHugh–Nagumo ring with electric inhibitory coupling: A reduced gradient description

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We study from the *nonequilibrium-potential* (NEP) viewpoint, the self-organization induced by local noises with common variance η—which includes synchronization with a global, periodic, adiabatic and subthreshold signal $S(t)$—of networks of excitable elements with FitzHugh–Nagumo (FHN) dynamics. After having coupled (chemically[1–3] or electrically[4]) their *activator* fields in *antiphase* fashion, we study electric coupling (in phase, with strength E) between nearest-neighbors' *inhibitor* fields. Excitability allows us to set a threshold u_{th} (its precise value is irrelevant as far as it lies within some interval), call cell i *active* if $u_i > u_{th}$ and define the (time-dependent) *activity* as the average number of active cells.

As in previous work[1–4], we study numerically the activity as a function of η for fixed E (Fig. 1), finding a stochastic-resonant behavior (Fig. 2), and elucidate the dynamics in terms of noise-assisted transitions between (signal-dependent) attractors. We introduce a *two-cell* model that allows the characterization of an antipahase state (besides homogeneous one). At variance with former cases[1–4], a NEP is not easily found for this system. So we project the dynamics along the *slow manifolds.* All the point-like attractors fall along this projection. This alternative is used *only* to calculate *heigths* of barriers. The resulting system is gradient, and its potential ϕ is symmetric under $u_1 \leftrightarrow u_2$.

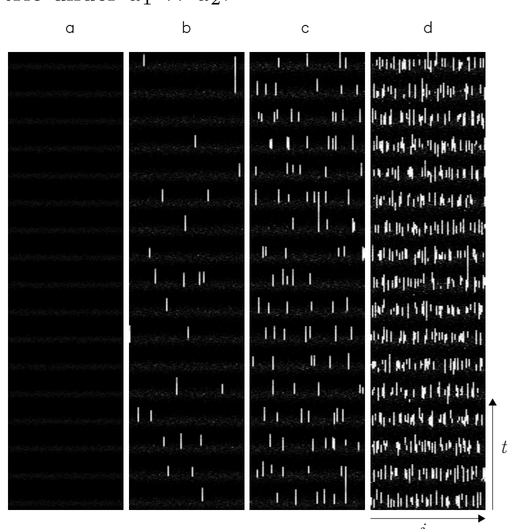

FIG. 1. Record of a set of 80 cells (white: 'activated', black: 'inhibited') with $E = 1$. a) $\eta = 2 \times 10^{-8}$ (subthreshold homogeneous oscilation), b) $\eta = 1.4 \times 10^{-7}$ (Partially synchronized state), c) $\eta = 2 \times 10^{-7}$ (partially synchronized state), and d) $\eta = 5.6 \times 10^{-7}$ (noise-sustained synchronization).

The number and relative stability of attractors changes with the signal amplitude (Fig. 3). For maximum amplitude, the difference in ϕ between the saddle and the uniform state is $\Delta\phi = 7 \times 10^{-7}$. *For a noise level of this order, the two-cell system would climb that potential barrier and transit to the antiphase state, that has a lower ϕ value.* The antiphase state can then *deterministically* return to the uniform state because the saddle-antiphase fixed points collapse. $\eta \sim \Delta\phi$ is the expected order of magnitude of noise for full synchronization, in good agreement with the numerical results.

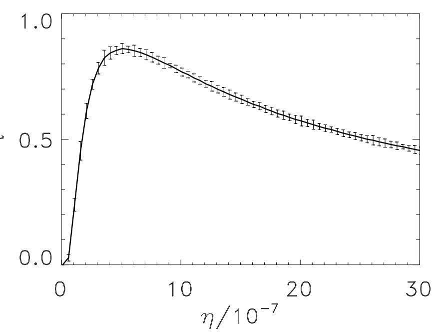

FIG. 2. $Q-$factor of the activity for $E = 0.5$ (average over 20 realizations). Q_{max} occurs at $\eta \approx 5 \times 10^{-7}$.

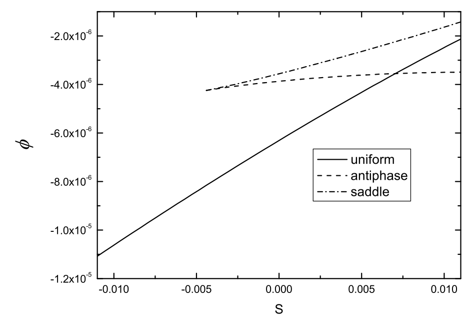

FIG. 3. ϕ values vs S for the uniform (inhibited-inhibited), the saddle and the antiphase states. Note that the last two fixed point disappear for S below $\sim -5 \times 10^{-3}$ (deterministic decay). Here $E = 0.5$

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