Transport coefficients of a granular gas of inelastic rough hard spheres

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The dynamics of a dilute granular gas, modeled as a system of hard spheres colliding inelastically with constant coefficients of normal (α) and tangential (β) restitution, can be described at a mesoscopic level by the (inelastic) Boltzmann equation for the one-body velocity distribution function $f(\mathbf{r},\mathbf{v},\boldsymbol{\omega},t)$.[1,2] A granular gas is intrinsically out of equilibrium and thus, in contrast to the case of energy conservation (elastic collisions), breakdown of equipartition is present (i.e., $\theta \equiv T^{\rm rot}/T^{\rm tr} \neq 1$, where $T^{\rm tr}$ and $T^{\rm rot}$ are the translational and rotational *granular* temperatures, respectively), even in homogeneous and isotropic states.

There exists compelling evidence on the applicability of hydrodynamics to granular gases. On the other hand, while the Navier–Stokes–Fourier (NSF) transport coefficients have been obtained from the Boltzmann equation in the case of *smooth* spheres[3] (i.e., $\beta=-1$), the generalization to rough spheres ($-1<\beta\leq 1$) is yet unclear.[4]

The aim of this work is to present the NSF constitutive equations for arbitrary values of α and β, as derived from application of the Chapman–Enskog method to the Boltzmann equation. In order to obtain explicit expressions, the usual (first) Sonine approximation is used. In the case of zero mean spin, the structure of the constitutive equations is

$$P_{ij}=\left(\frac{2nT}{1+\theta^{(0)}}-\eta_b\nabla\cdot\mathbf{u}\right)\delta_{ij} -\eta\left(\nabla_i u_j+\nabla_j u_i-\frac{2}{3}\nabla\cdot\mathbf{u}\delta_{ij}\right),$$

$$\mathbf{q}=-\kappa\nabla T-\mu\nabla n,$$

$$\zeta=\zeta^{(0)}+\xi\nabla\cdot\mathbf{u}.$$

Here, n is the number density, $T=\frac{1}{2}\left(T^{\rm tr}+T^{\rm rot}\right)$ is the total temperature, $\mathbf{u}$ is the flow velocity, P_{ij} is the pressure tensor, $\mathbf{q}$ is the total energy flux, and ζ is the cooling rate. The superscript (0) denotes quantities in the homogeneous cooling state, while η_b (bulk viscosity), η (shear viscosity), κ (thermal conductivity), μ, and ξ are NSF transport coefficients. In the smooth case $\eta_b=\xi=0$. Moreover, $\mu=0$ for elastic and smooth spheres.

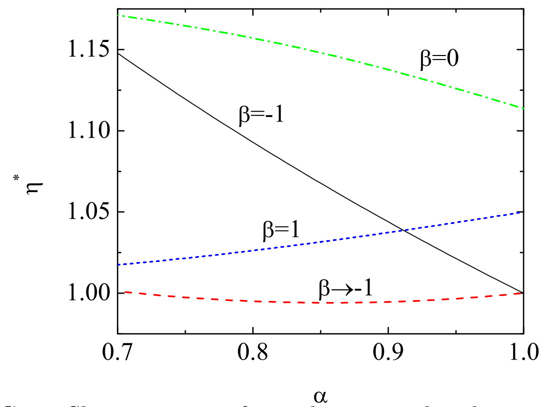

FIG. 1. Shear viscosity for inelastic rough spheres in the limit $\beta\to-1$, at $\beta=0$, and at $\beta=1$. The curve corresponding to smooth spheres ($\beta=-1$) is also included.

As an illustration, Fig. 1 shows the α-dependence of the shear viscosity (reduced with respect to that of elastic and smooth spheres) at several values of β. We observe that the limit $\beta\to-1$ widely differs from the result for pure smooth spheres ($\beta=1$). This is a consequence of the *singular* character of the limit $\beta\to-1$, in which case $\theta^{(0)}$ diverges and the influence of the internal degrees of freedom cannot be neglected.[2] In fact, the known results for smooth spheres[3] are recovered if, in addition to $\beta\to-1$, one *formally* makes $\theta^{(0)}\to 0$. In the special case of elastic and completely rough spheres ($\alpha=1$, $\beta=1$), Pidduck's expressions for the transport coefficients[5] are reobtained.

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[1] A. Santos, G. M. Kremer, and V. Garzó, *Prog. Theor. Phys. Suppl.* **184**, 31 (2010).
[2] A. Santos, G. M. Kremer, and M. dos Santos, *Phys. Fluids* **23**, 030604 (2011).
[3] J. J. Brey, J. W. Dufty, C. S. Kim, and A. Santos, Phys. Rev. E 58, 4638(1998).
[4] A. Goldshtein and M. Shapiro, J. Fluid Mech. 282, 75 (1995).
[5] G. M. Kremer, *An Introduction to the Boltzmann Equation and Transport Processes in Gases* (Springer, Berlin, 2010).