Smart Inference and Criticality

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There is increasing evidence that some aspects of living systems exhibit critical-like behavior[1]. One can find examples ranging from flock dynamics to regulatory genetic networks and neural avalanches in the brain. However, understanding why this happens and the mechanisms that bring individuals to exhibit criticality is still unclear.

To shed some light on such a general question, we present a different perspective to understand criticality, in the context of Information Theory[2]. Within this framework, we sketch some of the general aspects of living systems modeling them as *inferring* systems.

We implement two variants of our model, inspired by the genetic algorithm. In a nutshell, we have a community of individuals coping with an external environment, gathering the information coming from different sources. We find that the best strategy to do it efficiently is to tune their parameters in the vicinity of the critical point, provided that there is any. But even in the absence of any external sources, the same result still applies when individuals, far away from criticality, try to cope with each other. The critical point becomes a global attractor in response to this smart inference.

Mathematically, we encode a source of stimuli $\vec{s} = (s_1, ..., s_N)$ in a distribution $P_{\rm src}(\vec{s}|\vec{\alpha})$. Allowing parameters $\vec{\alpha}$ to vary, different sources can be modeled. On the other hand, each of the individuals in the community seeks to represent, with the largest possible fidelity, the essential aspects of the stimuli coming from the source in a distribution $P_{\rm map}(\vec{s}|\vec{\beta})$. Different values of the parameters $\vec{\beta}$ accounts for different maps. Finally, to measure the 'closeness' between both distributions, Information Theory provides a robust measure through the Kullback-Leibler divergence, $D_{KL}(\vec{\alpha}|\vec{\beta}) = \sum_{\vec{s}} P_{\rm src}(\vec{s}|\vec{\alpha}) \log \left(P_{\rm src}(\vec{s}|\vec{\alpha}) / P_{\rm map}(\vec{s}|\vec{\beta}) \right)$, which quantifies the loss of information when the map is used to approximate the source:

In the first model, we have a community of M individuals, each one characterized by its internal parameters $\vec{\beta}$. At every time step, S sources are generated at randomly from a pool $\rho_{\rm src}(\vec{\alpha})$. Then, we kill one of the individuals with a probability proportional to the mean KL divergence with respect to the environment,

$$P_{\rm kill}(i) \propto \frac{1}{S} \sum_{k=1}^{S} D_{KL}(\vec{\alpha}_k|\vec{\beta}_i), \quad i = 1, ..., M. \tag{1}$$

The unlucky individual is replaced by another one in the community, and then we introduce a perturbation in the parameters with a small probability.

When the process is iterated, if the internal map presents a continuous phase transition, and *only* when the environment $\rho_{\rm src}$ is highly heterogeneous, parameters fall down in the vicinity of the critical point, $\vec{\beta} = \vec{\beta}_c$.

In the second model, there is no *a priori* choice for the pool of sources, and the community plays itself the role of the environment. In other words, individuals evolve trying to 'understand' each other. At every time step, S individuals are picked at randomly, and one of them is killed with a probability proportional to the mean D_{KL} respect to the other $(S-1)$ individuals. Mutations are introduced again with a small probability.

In this case, and regardless of the initial conditions, complexity emerges and individuals' parameters tune themselves in the critical point.

In summary, the observed criticality in living systems can be understood as a result from the necessity of coping with many diverse external conditions and their ability of building up accurate maps of the environment.

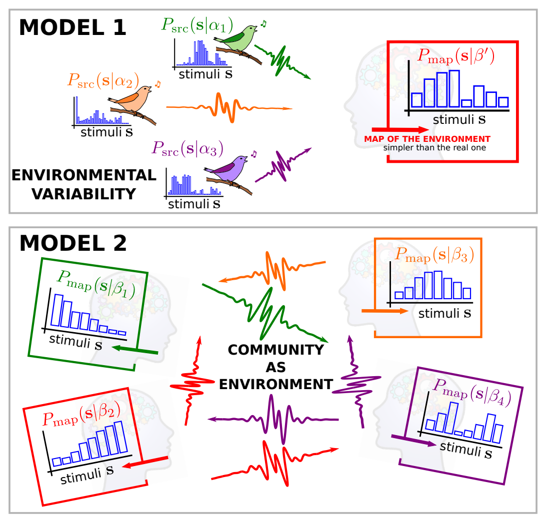

FIG. 1. Sketch of the two computational models.

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