Stretched criticality and localization in hierarchical networks

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The nature of the underlying contact pattern is known to play a crucial role in spreading and synchronization phenomena. The ability to access real data has recently revealed that several systems of biological and techno-social relevance can be described as hierarchical modular networks (social networks, brain networks, gene regulatory networks etc.). Hierarchical modular networks may exhibit fractal-like properties across several levels and consequently give rise to exotic dynamic behavior.

We study dynamical models of spreading, ordering and synchronization in hierarchical modular networks. We show that the traditional critical-point scenario, which characterizes spreading phenomena in generic complex networks, is replaced by an extended critical-like region, a Griffiths phase, which stems from the network structural multi-level heterogeneity [1]. Throughout the Griffiths phase, the system exhibits the attributes usually associated with critical points, such as enhanced response and huge dynamic range. More importantly, the Griffiths phase is characterized by persistent activity, lingering in rare regions that remain active for exponentially large times even below the dynamic threshold. Recent studies have shown how network heterogeneity or disorder may give rise to Griffiths phases in complex networks [2]. In analogy with the phenomenon of Anderson localization in disordered media, we prove that the emergence of the Griffiths phenomenon is related to the existence of localized states in the network contact pattern, resulting in exteded regions of persistent activity and slow synchronization. We provide a numerical method for fast sampling of the spectral hallmarks of localization in the adjacency matrix of large networks and propose a revised theoretical framework that adds localization to the traditional mean-field description.

Our methods prove effective in the study of activity in brain networks, where the extended regions of criticality observed in experiments arise from Griffiths phases induced by the hierarchical architecture of the brain neural network [3]. More importantly, we believe that our results on rare regions of localized and persistent activity are of great relevance to spreading processes occurring in low-dimensional disordered systems as well as hierarchical and localized contact patterns in social sciences and epidemiology, whether they are induced by social factors, non-trivial mobility constraints, geographical isolation or targeted border policies.

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