Strong anisotropy in two-dimensional surfaces with generic scale invariance: Non-linear effects

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Scale invariant, two-dimensional surfaces that are anisotropic in space abound in Science and Technology, for systems spanning many orders of magnitude in length scales. Examples range from epitaxial thin films in nanoscience[1] to micro and macroscopic crack formation in solids,[2] to geological systems, such as landscape evolution induced by rivers.[3] Mathematically, the surfaces that occur in these and many other systems are self-affine fractals,[4] whose fractal dimension (or, equivalently, roughness exponent) differs, depending on the direction along which it is measured. Due to the lack of characteristic distances, the scaling behavior just described is a form of anisotropic critical behavior,[5] which moreover often occurs without the need of parameter fine-tuning that adjusts the system to a critical point. These are thus examples of so-called generic scale invariance (GSI).[6] A context for this type of behavior, in which anisotropy has remained relatively little studied, is that of surface kinetic roughening.[4] In this work we pursue a continuum description of GSI systems through stochastic partial differential equations. Our cases of interest will be those conditions that lead to GSI while applying to the most important universality classes in surface kinetic roughening. Namely,[6] systems with non-conserved dynamics, like the celebrated Kardar-Parisi-Zhang (KPZ) equation,[7] or else systems with conserved dynamics and non-conserved noise, like e.g. the so-called conserved KPZ (cKPZ) equation.[8] Remarkably, the anisotropic generalizations of the two previous equations, namely, the so-called anisotropic KPZ (aKPZ)[9] and conserved anisotropic KPZ (caKPZ) equations,[10] do *not* lead asymptotically to anisotropic behavior (strong anisotropy, SA). Rather, in spite of being nominally anisotropic, they lead to isotropic asymptotics (weak anisotropy, WA), in universality classes that depend on parameter conditions. This fact contrasts strikingly with the unambiguous observation of SA in experiments on surface kinetic roughening for two-dimensional interfaces, see Ref. 11 and references therein.

In this work we focus on a number of representative equations, like the Hwa-Kardar equation, proposed in the context of self-organized criticality,[12] and both the conserved and non-conserved anisotropic KPZ equations.[9,10] All of them display GSI, and remained outside our previous analysis,[13] due to the unavailability of accurate approximations through linear equations for most of the cases. Thus, here we employ techniques that in principle can tackle strongly non-linear systems, such as the Dynamic Renormalization Group and direct numerical simulations. We show[14] that for non-conserved dynamics SA simply does not occur, even for special conditions under which only one of the nonlinearities is suppressed. On the other hand, for systems with conserved dynamics SA can be obtained, and even whole families of equations can be formulated which display this property. However, both in the presence and in the absence of the shift symmetry $h \to h + \text{const.}$, this seems only possible for "incomplete" equations in which one of the nonlinearities is suppressed. In general, conditions of this type depend critically on details of the dynamics that is being described, acting as special constraints, and are in this sense non-generic in parameter space. Hence, they cannot be obtained from simple-minded derivations of the equations of motion based on symmetries and conservation laws.

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[1] C. Misbah, O. Pierre-Louis, and Y. Saito, Rev. Mod. Phys. **82**, 981 (2010).
[2] D. Bonamy and E. Bouchaud, Phys. Rep. **498**, 1 (2011).
[3] R. Pastor-Satorras and D. H. Rothman, J. Stat. Phys. **93**, 477 (1998).
[4] A.-L. Barabási and H. E. Stanley, *Fractal Concepts in Surface Growth*, Cambridge University Press, Cambridge, UK (1995).
[5] U. C. Täuber, *Critical dynamics: a field theory approach to equilibrium and non-equilibrium scaling behavior*, unpublished. *http://www.phys.vt.edu/tauber.28*
[6] G. Grinstein, *Scale Invariance, Interfaces, and Non-Equilibrium Dynamics*, Plenum Press, New York (1995).
[7] M. Kardar, G. Parisi, and Y.-C. Zhang, Phys. Rev. Lett. **56**, 889 (1986).
[8] Z.-W. Lai and S. Das Sarma, Phys. Rev. Lett. **66**, 2348 (1991).
[9] D. E. Wolf, Phys. Rev. Lett. **67**, 1783 (1991).
[10] H. Kallabis, J. Phys. A: Math. Gen. **31**, L581 (1998).
[11] E. Vivo, M. Nicoli, M. Engler, T. Michely, L. Vázquez, and R. Cuerno, Phys. Rev. B **86**, 245427 (2012).
[12] T. Hwa and M. Kardar, Phys. Rev. Lett. **62**, 1813 (1989).
[13] E. Vivo, M. Nicoli, and R. Cuerno, Phys. Rev. E **86**, 051611 (2012).
[14] E. Vivo, M. Nicoli, and R. Cuerno, arXiv:1311.7638 (2013).