Burnett transport coefficients for inelastic Maxwell models

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The inelastic Maxwell model (IMM) is used to describe a d-dimensional granular gas of particles of mass m. In the IMM, the one-particle distribution function obeys a Boltzmann-like equation with the usual hard-sphere collision rate replaced by a velocity-independent one. The effective frequency ν_0 is taken to be proportional to the density of particles n and the γ-power of the temperature T, i.e. $\nu_0 \propto nT^\gamma$. In that way, different values of γ can be chosen to mimic different interaction potentials in the elastic limit. In particular, $\gamma = 1/2$ represents hard spheres. The modification of the operator allows for explicit calculations[1] and, in general, leads to reasonable results[2].

In this work, the kinetic equation of the IMM is solved by means of the Chapman-Enskog method up to the second order in the spatial gradients of the hydrodynamic fields (density of particles n, macroscopic velocity $\mathbf{u}$, and temperature T). As a consequence, the constitutive equations for the pressure tensor and the heat flux are evaluated up to Burnett order, and hence a closed hydrodynamic description up to third order in the gradients is obtained.

In the Burnett order, the constitutive relations for the pressure tensor is

$$
\begin{aligned}
P_{ij}^{(2)} = {} & a_1 \frac{\kappa_0}{\nu_0}\left(\nabla_i\nabla_j T - \frac{1}{d}\delta_{ij}\nabla^2 T\right) \\
& + a_2 \frac{T\kappa_0}{p\nu_0}\left(\nabla_i\nabla_j p - \frac{1}{d}\delta_{ij}\nabla^2 p\right) \\
& + a_3 \frac{\kappa_0}{T\nu_0}\left[\nabla_i T\nabla_j T - \frac{1}{d}\delta_{ij}(\nabla T)^2\right] \\
& + a_4 \frac{T\kappa_0}{p^2\nu_0}\left[\nabla_i p\nabla_j p - \frac{1}{d}\delta_{ij}(\nabla p)^2\right] \\
& + a_5 \frac{\kappa_0}{p\nu_0}\left(\nabla_i T\nabla_j p + \nabla_i p\nabla_j T - \frac{2}{d}\delta_{ij}\nabla p\cdot\nabla T\right) \\
& + a_6 \frac{\eta_0}{\nu_0} D\left(D_{ij} - \frac{1}{d}\delta_{ij} D\right) \\
& + a_7 \frac{\eta_0}{\nu_0}\Big[D_{ik}D_{kj} - \omega_{ik}\omega_{kj} \\
& \quad - \frac{1}{d}\delta_{ij}\left(D_{lk}D_{kl} - \omega_{lk}\omega_{kl}\right) + \omega_{ij}D_{kj} - D_{ik}\omega_{kj}\Big],
\end{aligned}
$$

where $D = \nabla\cdot\mathbf{u}$, $D_{ij} = \frac{1}{2}(\nabla_i u_j + \nabla_j u_i)$, $\omega_{ij} = \frac{1}{2}(\nabla_j u_i - \nabla_i u_j)$, κ_0 is the elastic thermal conductivity, η_0 is the elastic viscosity, $p = nT$ is the pressure, and a_i are reduced transport coefficients that depend on d, the coefficient of normal restitution α, and γ, see Fig. 1. For the heat flux

$$
\begin{aligned}
q_i^{(2)} = {} & b_1 \frac{T\kappa_0}{\nu_0}\nabla^2 u_i + b_2 \frac{T\kappa_0}{\nu_0}\nabla_i D + b_3 \frac{\kappa_0}{\nu_0} D_{ij}\nabla_j T \\
& + b_4 \frac{\eta_0}{\rho\nu_0} D_{ij}\nabla_j p + b_5 \frac{\kappa_0}{\nu_0}\omega_{ij}\nabla_j T + b_6 \frac{\eta_0}{\rho\nu_0}\omega_{ij}\nabla_j p \\
& + b_7 \frac{\kappa_0}{\nu_0} D\nabla_i T + b_8 \frac{\eta_0}{\rho\nu_0} D\nabla_i p,
\end{aligned}
$$

where $\rho = nm$ is the mass density, and b_i are transport dimensionless coefficients. Figure 1 displays the α-dependence of the Burnett coefficients for $d = 3$ and $\gamma = 1/2$.

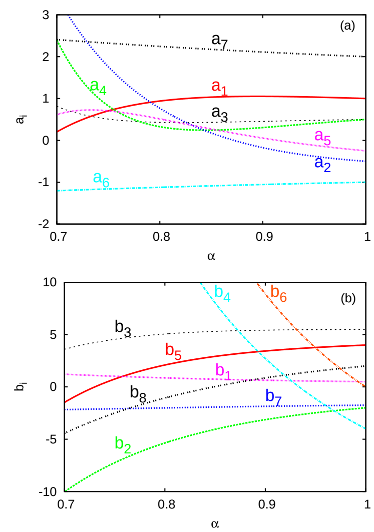

FIG. 1. Coefficients a_i (a) and b_i (b) for a three dimensional system ($d = 3$) and for $\gamma = 1/2$, as a function of the coefficient of normal restitution α.

For the elastic case ($\alpha = 1$), and for a three dimensional system, the expressions for the pressure tensor and the heat flux reduce, for any interaction potential between particles, to the classical ones in Ref.[3]. In addition, we provide a generalization of the elastic results to any dimension and any degree of dissipation. For inelastic situations, the structure of the constitutive equations is more complex than in the elastic case.

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